The following is based on a 3-hour lecture on "**Monte Carlo Event Generators**" given at the summer school "Heavy Ion Collisions in the QCD phase diagram", June 27 - July 08, 2022, Nantes, France.

It discusses elements of Monte Carlo event generators in general, but in particular the basic principles of parallel scatterings and their realisation in the new **EPOS4** scheme.

Monte Carlo Event Generators

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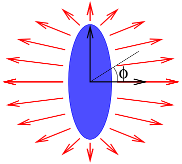
Contents

1 Introduction

1.1 Challenges

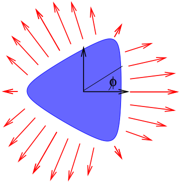
Since 2 decades we know: Colliding heavy ions at relativistic energies

behave like an expanding fluid, with huge transverse flow



(observables: pt spectra)

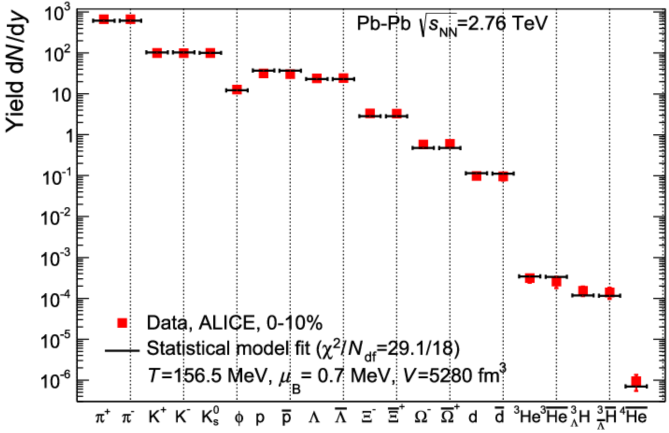
being in particular asymmetric: elliptical / triangular ...



(observables: flow harmonics v2, v3 etc)

We see "statistical particle production"

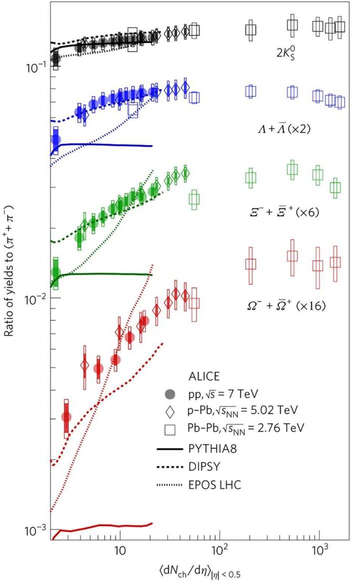
(observables: particle yields or ratios)



A Andronic et al 2017 J. Phys.: Conf. Ser. 779 012012

Very different compared to particle production from string decay

But similar features show up in small systems, at low energies, and as well for heavy flavor particles.



Yields/pions vs multiplicity, for pp, pPb, PbPb
(ALICE, in nature physics 2017)

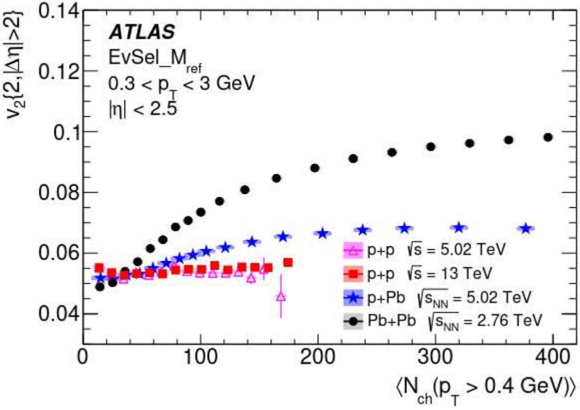
Central PbPb understood as due to "statistical particle production"

But it seems that pp and pPb are at least partly also showing this behavior

The event generators ... clearly need to be improved

v2 vs multiplicity for pp, pPb, PbPb

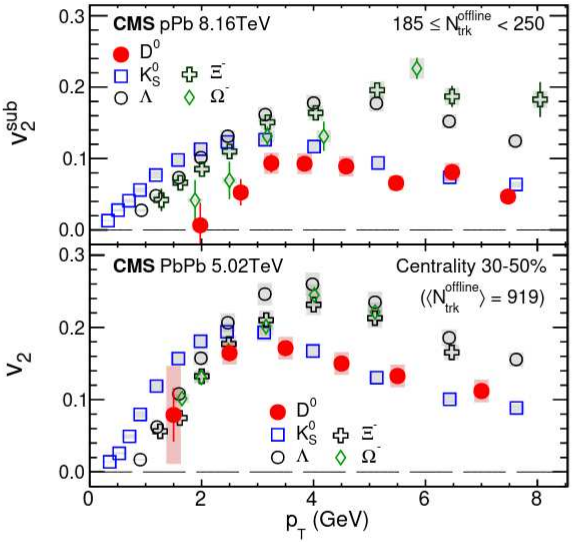
(Eur. Phys. J. C 77 (2017) 428)



Large v2 values (flow) for all systems, but different N_ch dependence

Small energy dependence

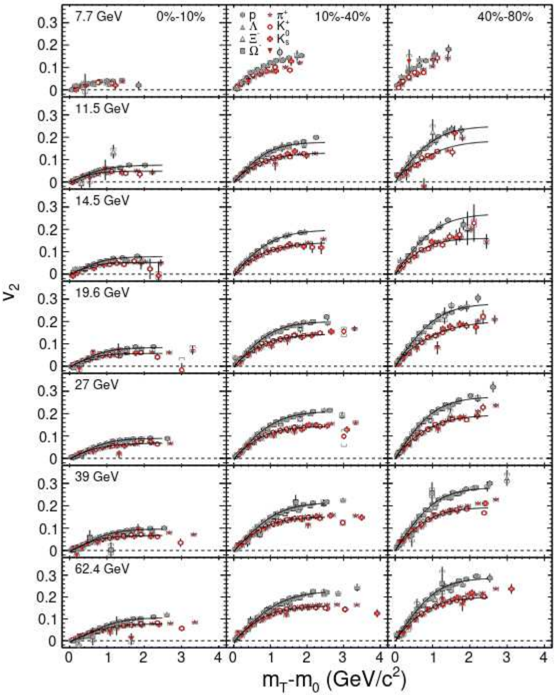
small N_ch dependence in pp



v2 vs pt
for pPb at 8.16TeV
and PbPb at 5.02TeV
(Phys. Rev. Lett. 121, 082301)

Large v2 values in pPb even for D mesons

Similar to K_s at large pt ("usual" meson behavior)



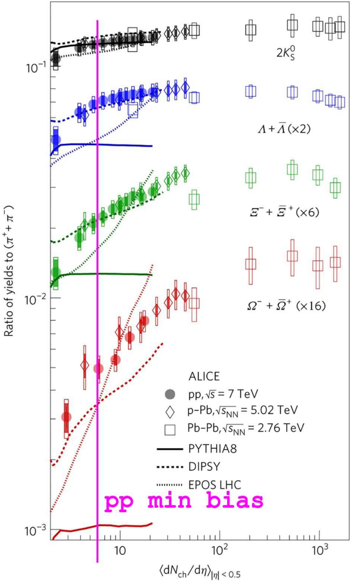
v2 vs m_T

for AuAu at 7-62 GeV

(Phys.Rev.C 93 (2016) 1, 014907)

Similar behavior down to low energies (where no QGP is expected)

Actually flow / statistical decay issues are relevant even for min bias pp!



elementary pp models
(particle production simply based on string decay)

do not produce enough Ω baryons even for min bias pp

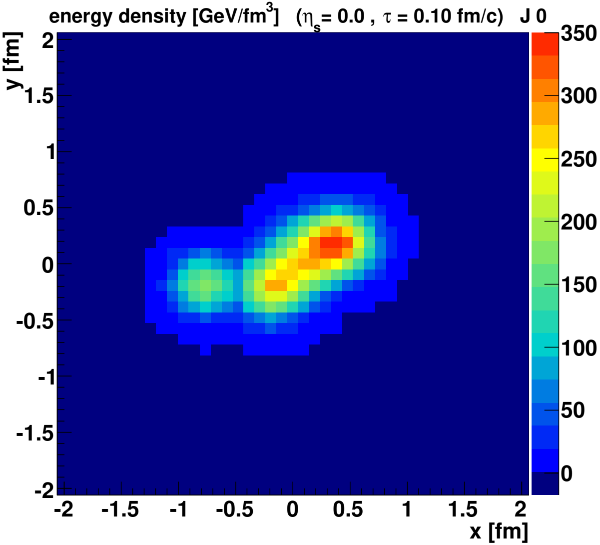
so some "new input" is needed ... compatible with the "normal" pp behavior (jets etc)

So these "features" (flow, stat. hadronizaton,...), **usually referred to as "QGP signals", expected in high energy heavy ion collisions,**

- **show up in pp scattering, even min bias**
- **show up in "low energy" collisions**
- **concern even charmed hadrons**

In particular the "small systems" (pp, pA) are very interesting...

EPOS simu pp 7TeV



Energy density vs x,y

Tiny

Very short lived (<2fm/c)

Very energetic here 350 GeV/fm3

(nuclear matter: 0.16 GeV/fm3)

Very strongly interacting

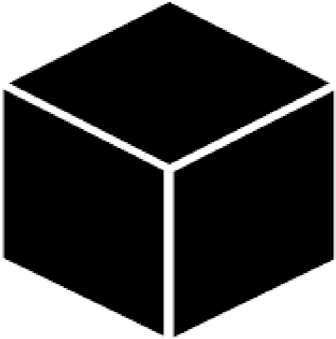
(fluid-like)

We better understand all that in a quantitative fashion ... and not to forget high pt features happening at the same time! We have

- ☐ **these “mini-plasmas” producing low pt particles**
  (soft domain)
- ☐ **and very high pt particles**
  (from pQCD processes, hard domain)

=> we need general purpose Monte Carlo Event Generators which allow to incorporate and test these “features”

1.2 What means "Monte Carlo Method"



It should NOT be a black box producing "events" of particles

to be compared with "real" events

Monte Carlo Method means

- □ **a tool to solve well defined mathematical problems**
- □ **based on probability theory**

  (random variables and random numbers)

Example: Compute $I = \int_0^1 f(x)dx$, which may be written as

$$I = \int_{-\infty}^{\infty} w(x)\, f(x)\, dx, \text{ with } w(x) = \begin{cases} 1 & \text{for } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

We may interprete w as probability distribution and I as expectation value (or mean value), so

$$I = \langle f \rangle = \underbrace{\frac{1}{N} \sum_{i=1}^{N} f(x_i)}_{MC\ estimate} + O\left(\frac{1}{\sqrt{N}}\right)$$

with uniform (in [0,1]) random numbers x_i

An error of order $1/\sqrt{N}$ is huge, nobody computes an 1D-integral like that, BUT for computing high-dimensional integrals, the formula

$$I = \int w(x_1, ..., x_n)\, f(x_1, ..., x_n) dx_1 ... dx_n$$

$$= \underbrace{\frac{1}{N}\sum_{i=1}^{N} f(x_{1i}, ..., x_{ni})}_{MC\ estimate} + O\left(\frac{1}{\sqrt{N}}\right)$$

is very useful.

Attention: MC sums over N "events", but these MC events are not necessarily "physical" events

So, Monte Carlo Method (as discussed in this talk) means more precisely

- □ **a tool to compute integrals $\int w(X)\, f(X)dX$ of a multi-dimensional variable X**
- □ **as mean value $\langle f(X)\rangle$ with X distributed according to w (with w being a multi-dimensional distribution)**

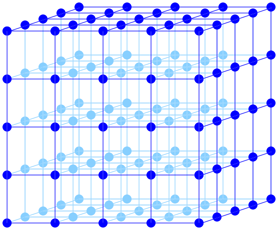
1.3 Monte Carlo Methods and the Ising Model

Actually generating n-dimensional X distributed according to some given w is usually very complicated for large n

- **a problem well known in statistical physics since a long time**

- **with intelligent solutions**

Extremely useful: The Ising model of ferromagnetism



Box of $N \times N \times N$ atoms each one carrying a spin with possible values +1 and -1 (spin up, spin down)

- ☐ **Anyhow useful to know, one deals with phase trasitions very similar to the QGP phase transition**
- ☐ **The MC methods used there are precisely what we need for heavy ion simulations**
- ☐ **Good example of a multi-dimensional variable X, being here the N^3 spin values, let us call it a "state"**

The interesting quantity here is the average magnetization $\langle M \rangle$:

$$\langle M \rangle = \sum w(X)\, M(X)$$

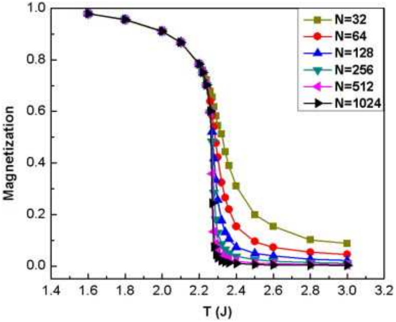
with

$$w(X) = \frac{1}{Z}\, e^{-\beta E(X)}$$

with

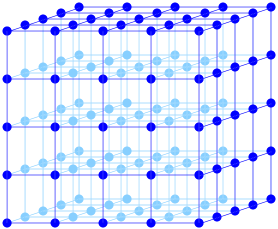
$$E = -\alpha \sum_{\text{neighbors } k,k'} s_k s_{k'}$$

Phase transition
2D Ising model



Journal of Computational Physics, 237, (2013) 224

Why difficult?



For N^3 atoms, the number K of possible states is $2^{\left(N^3\right)}$
$N = 100 : K \approx 10^{300000}$

Solution: Monte-Carlo method :

$$\langle M\rangle = \sum_{i=1}^{K} w(X_i)\, M(X_i) \quad \rightarrow \quad \frac{1}{J}\sum_{j=1}^{J} M(X_j)$$

with "reasonable" J, and X_j distributed according to $w(X)$

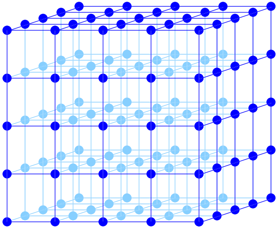
... provided we know how to generate X according to w(X)

1.4 Ising Model and Markov chains

The problem is:
generate a "state" X according to

$$w(X) = \frac{1}{Z}\, e^{-\beta E(X)}$$

corresponding to "themal equilibrium"



X is one of the $2^{\left(N^3\right)}$ possible states of the lattice

Simple "direct methods" (rejection sampling) do not work.

Idea: Let's copy nature which always finds eventually the "equilibrium distribution"

One considers a stochastic iterative process (Markov chain)

$$w_1 \rightarrow w_2 \rightarrow ...$$

A. Markov

with appropriate transitions $w_t \rightarrow w_{t+1}$ (Metropolis) such that w_t converges to $w_\infty = \frac{1}{Z}\, e^{-\beta E(X)}$

(it works, thanks to "fixed point theorems")

Why useful for us ?

- **Markov chain + Metropolis is extremely powerful,**
  it works for ANY distribution and not just Boltzmann distributions

- **It allows to treat "parallel interactions" in high energy scattering**

- **We use it for microcanonical QGP decay (needed for small systems)**

1.5 Parallel and sequential scattering

Some crucial thoughts about the validity of certain theoretical concepts in AA and pp scattering

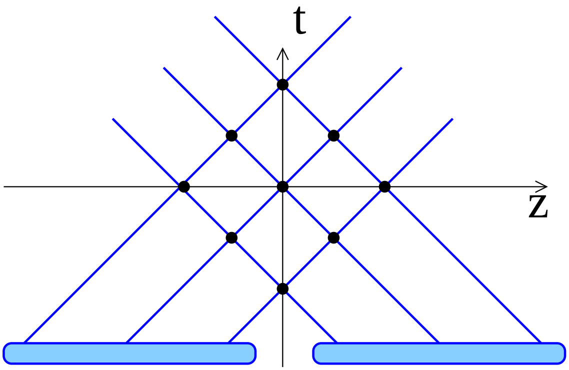
Parallel and sequential scattering in AA

Crucial time scales

$\tau_{\mathrm{collision}}$ **is the duration of the AA collision**

$\tau_{\mathrm{interaction}}$ **is the time between two NN interactions**

τ_{form} **is the particle formation time**



A+A collision in space-time
Blue lines: nucleons

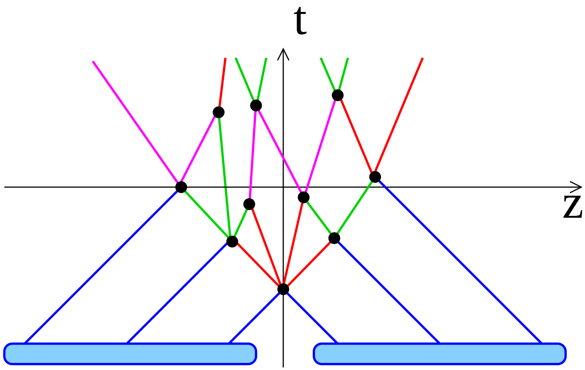
At "low" energy

Sequential collisions (cascade)

Crucial:

$\tau_{\text{form}} < \tau_{\text{interaction}}$

τ_{form} is the particle formation time
$\tau_{\text{interaction}}$ is the time between two NN interactions



A+A collision at low energy
red/green/magenta lines: produced hadrons

At "high" energy ($\gg$1GeV):

Longitudinal size

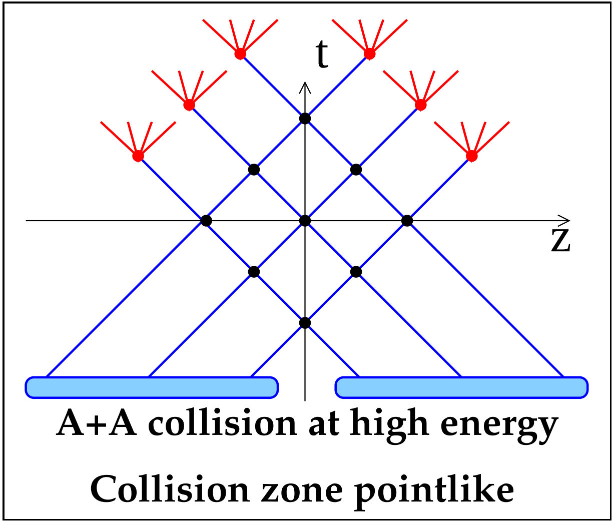
$$d = \frac{2R}{\gamma} \ll 1\,\mathrm{fm/c}$$

All interactions simultaneously at $t = 0$ (in parallel)

Particle production later. Condition:

$$\tau_{\mathrm{form}} \gg \tau_{\mathrm{collision}}$$

$\tau_{\mathrm{collision}}$ is the duration of the AA collision



Low energy and high energy nuclear scattering are completely different, and completely different theoretical methods are needed

- **High energy approach = parallel interactions (as done in EPOS)**

  (and this is why we need these Markov chain techniques...)

- **At LHC energies, one can completely separate**
  - **primary interactions (within < 0.01 fm/c)**
  - **and secondary interactions (hydro evolution etc)**

What is the range of validity of the "parallel approach" ?

The condition is

$$\tau_{\mathrm{collision}} = \frac{2R}{\gamma c} < \tau_{\mathrm{form}} \approx 1\,\mathrm{fm/c}$$

For $R = 6.5\,\mathrm{fm}$, we get

$$\gamma > \frac{2R}{c\tau_{\mathrm{form}}} \approx \frac{13}{1}$$

so the critical energy per nucleon is $E \approx 13\, m_p c^2 \approx 12\,\mathrm{GeV}$

The "parallel approach" is valid (and required) for $\sqrt{s_{NN}} \gtrsim 24\,\mathrm{GeV}$ (upper BES energies, LHC)

What is the range of validity of the “cascade approach” ?

The condition is (with n nucleons in a row)

$$\tau_{\text{interaction}} = \frac{2R}{n\gamma\beta c} > \tau_{\text{form}} \approx 1\,\text{fm/c}$$

For $R = 6.5\,\text{fm}$ and $n = 6$, we get

$$\gamma\beta < \frac{2R}{nc\tau_{\text{form}}} \approx \frac{13}{6}$$

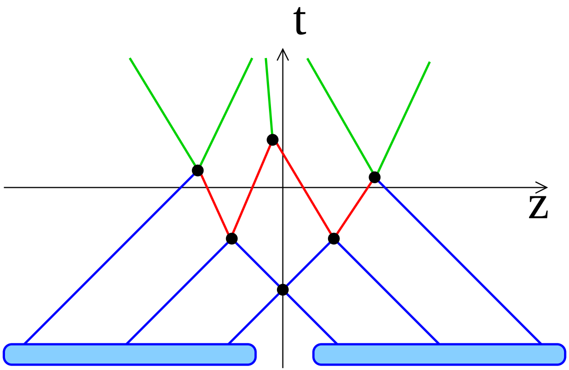
so the critical energy per nucleon is $E \approx \gamma\, m_p c^2 \approx 2.2\,\text{GeV}$

The “cascade approach” is valid for $\sqrt{s_{NN}} \lesssim 4\,\text{GeV}$

The intermediate range $4 < \sqrt{s_{NN}} < 24$ GeV

On needs a "partially parallel approach"

Several (but not all) NN scatterings are realized, before particle production starts



A+A collision at intermediate energy

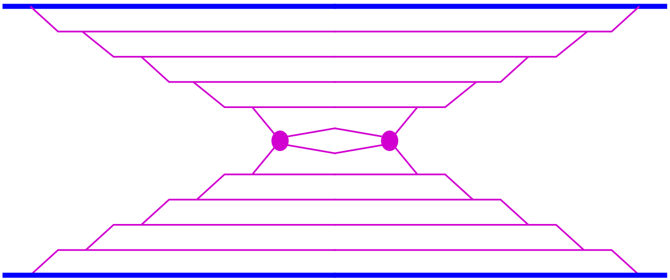
Red/green lines: produced hadrons

Parallel approach in pp

At LHC energy: Interaction: successive parton emissions

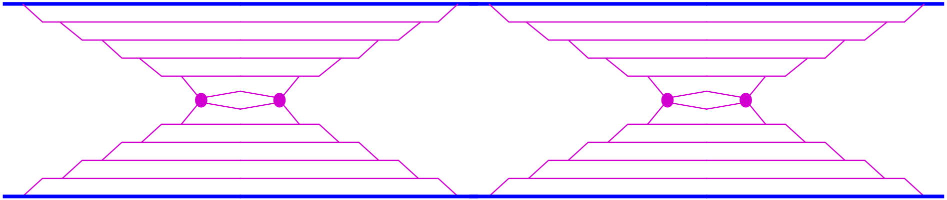
Large gamma factors, very long lived ptls

The complete process takes a very long time



pp collision
Exchange of parton ladder

Impossible to have several of these interactions in a row

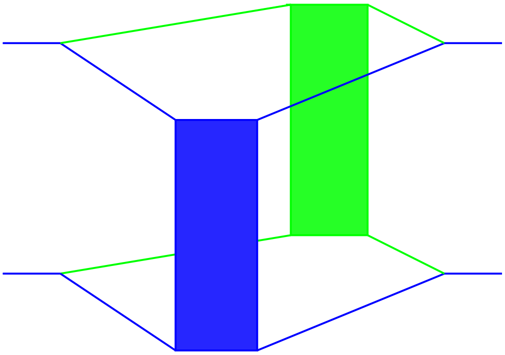


So also in pp:

- **High energy approach = parallel interactions (as done in EPOS)**

And we know that multiple scattering is important!

So double scattering in pp should look like this:

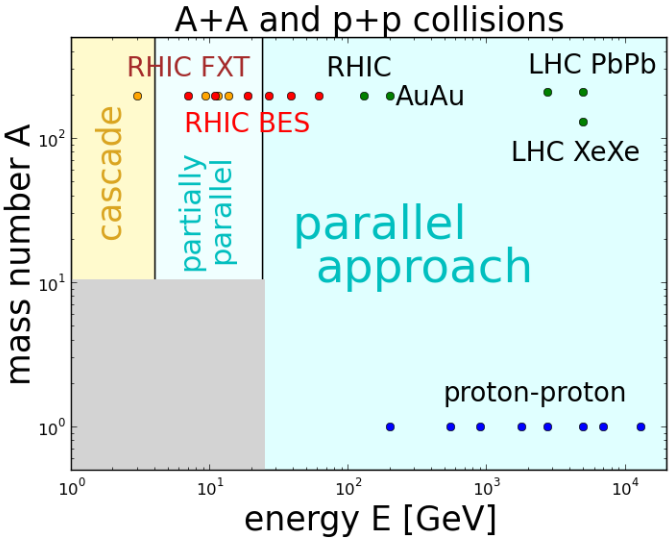


Here two parallel scatterings

No contradictions with respect to timescales

So it seems mandatory to use a parallel scattering scheme, for pp and AA, known since a long time ... but somewhat forgotten nowadays ...

Parallel approach needed almost everywhere



Points (besides FXT): Epos comparisons to data

1.6 Factorization and S-matrix

Factorization

The most popular approach to treat HE pp, is based on "factorization", where the di-jet cross section is given as

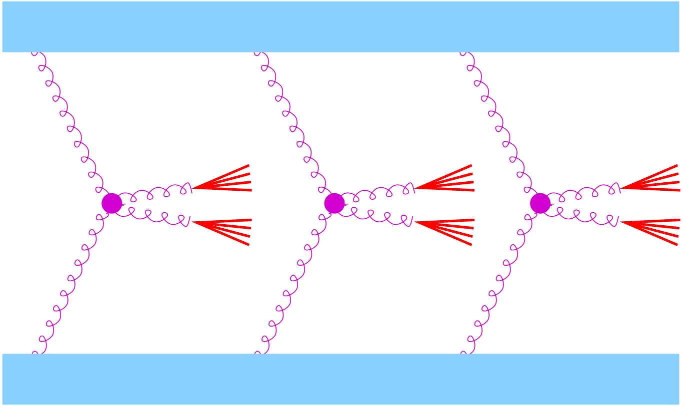
$$\sigma_{\rm dijet} = \sum_{kl} \int \frac{d^3p_3 d^3p_4}{E_3E_4} \int dx_1 dx_2\, f^k_{\rm PDF}(x_1,\mu_F^2)\, f^l_{\rm PDF}(x_2,\mu_F^2)$$
$$\frac{1}{32s\pi^2} \bar{\sum} |\mathcal{M}|^2 \delta^4(p_1+p_2-p_3-p_4),$$

Easy! No sophisticated MC needed.

But where are these complicated "parallel" scatterings?

The di-jet cross section is an inclusive cross section, i.e. one counts di-jets, not di-jet events, so a *N*-di-jet event counts *N* times

Here: N=3: We count 3 dijets.



Summing N-di-jet events, we get for the inclusive di-jet cross section

$$\sigma_{\mathrm{dijet}} = \sum_{N} N\, \sigma_{\mathrm{dijet}}^{(N)}$$

whereas the total cross section (forgetting soft for the moment)

$$\sigma_{\mathrm{tot}} = \sum_{N} \sigma_{\mathrm{dijet}}^{(N)}$$

For inclusive cross section, enormous simplifications apply!

To understand this we have to first look closer at "parallel scattering", using an appropriate tool (S-matrix approach).

Crash course on S-matrix theory

S-matrix theory is based on two major beliefs:

- **Even when a theory is not 100% known, one may obtain considerable guidance from a general quantum mechanical framework based on (plausible) hypotheses**

- **Properties of functions *f(x)* of real variables are much better understood when “continuing” into the complex plane** *)

---

*) based on the uniqueness of analytic continuation in the complex plane, an extremely power theorem

Reminder: The scattering operator $\hat{S}$ is defined via

$$|\psi(t=+\infty)\rangle = \hat{S}\,|\psi(t=-\infty)\rangle$$

The S-matrix is the corresponding representation

$$S_{ij} = \langle i|\,\hat{S}\,|j\rangle$$

for basis states $|i\rangle$ and $|j\rangle$.

The T-matrix is defined as

$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta(p_f - p_i)T_{fi}$$

Fundamental properties of the S-matrix

Most important:

The scattering operator $\hat{S}$ must be unitary:

$$\hat{S}^\dagger \hat{S} = 1$$

(elementary quantum mechanics), which means the scattering does not change the normalisation of a state.

Very plausible 3 hypotheses:

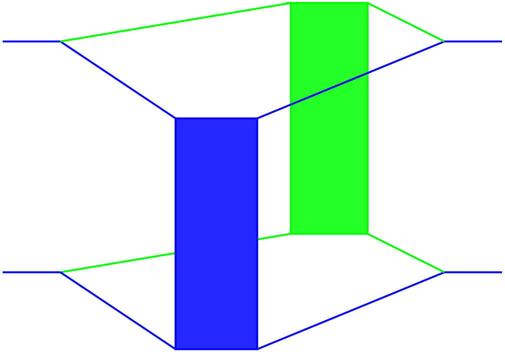
- ☐ T_{ii} is Lorentz invariant $\rightarrow$ use s, t
- ☐ $T_{ii}(s,t)$ is an analytic function of s, with s considered as a complex variable (Hermitean analyticity)
- ☐ $T_{ii}(s,t)$ is real on some part of the real axis

Using the Schwarz reflection principle (a theorem), **$T_{ii}(s,t)$ first defined for Im$s \geq 0$ can be continued in a unique fashion via $T_{ii}(s^*,t) = T_{ii}(s,t)^*$.**

In the following we use $T = T_{ii}$ (elastic scattering).

Based on the observation of multiple scattering at HE, and the necessity of parallel scatterings, one postulates a particular form of the (elastic scattering) **T-matrix** *):

$$-i\,\{iT_{\mathrm{Pom}} \times \ldots \times iT_{\mathrm{Pom}}\}$$



The elements are called Pomerons (just a name)

Compatible with pQCD, hidden in the "boxes" (Pomerons)

*) simplified version, without energy conservation

It can be shown (from unitarity + 3 hypotheses):

$$2s\,\sigma_{\mathrm{tot}} = (2\pi)^4 \delta(p_f - p_i) \sum_f \left|T_{fi}\right|^2 = \frac{1}{\mathrm{i}} \mathrm{disc}\, T$$

> **Interpretation: $\frac{1}{\mathrm{i}}\mathrm{disc}\, T$ can be seen as a so-called "cut diagram", with modified Feynman rules, the "intermediate particles" are on mass shell.**

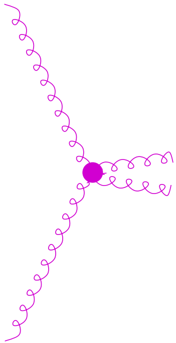
Cut diagrams ($\frac{1}{\mathrm{i}}\mathrm{disc}\, T$) represent inelastic processes, uncut diagrams (T) elastic ones.

The notion of "cutting" is extremely useful in our approach, more details later.

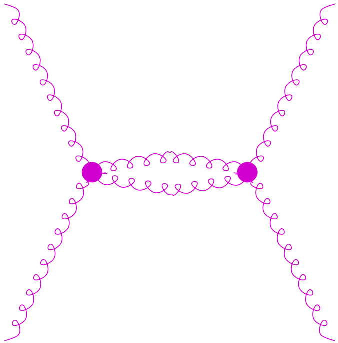
Example: Di-jet diagram

(We consider here jet = parton)

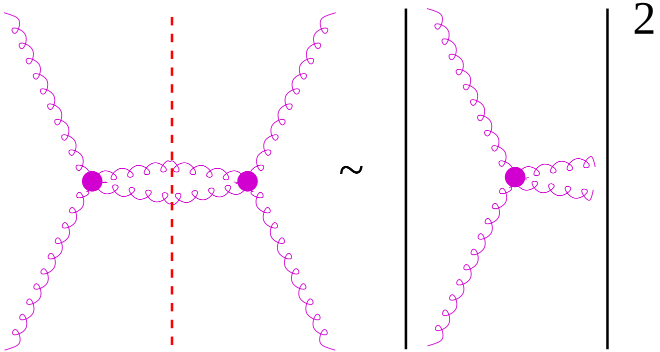
Inelastic diagram:



Elastic diagram:



The cut diagram is (up to constant) equal to squared inelastic one



The cut diagram ($\frac{1}{i}$disc T) represents inelastic processes, for T representing the corresponding elastic ones.

Why does factorization work ?

Easy to see in our **S-matrix approach** based on parallel scatterings (Gribov-Regge picture, as used in EPOS).

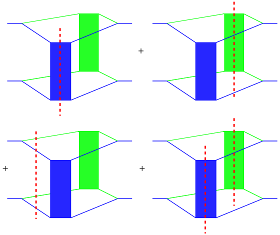
Here, the simplified version, without energy conservation, using simple assumptions:

Consider multiple scattering amplitude, i.e. a T-matrix of the form

$$iT = \prod iT_{\mathrm{P}}$$

T_P represents one elementary scattering (Pomeron)

Cross section: sum over all cuts.



Here, two parallel scatterings

A cut Pomeron is (up to a constant) equal to the inelastic amplitude squared, it represents the weight to produce a di-jet.

An uncut Pomeron is just an elastic scattering, nothing produced.

For each cut Pom:

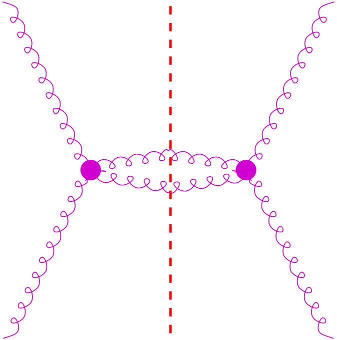
$$\frac{1}{i}\mathrm{disc}T_{\mathrm{P}} = 2\mathrm{Im}T_{\mathrm{P}} \equiv G$$

For each uncut one (considering imaginary T_{P}):

$$\begin{aligned} & iT_{\mathrm{P}} + \{iT_{\mathrm{P}}\}^{*} \\ = & \; i\,(i\,\mathrm{Im}T_{\mathrm{P}}) + \{i\,(i\,\mathrm{Im}T_{\mathrm{P}})\}^{*} \\ = & \; -2\mathrm{Im}T_{\mathrm{P}} \equiv -G \end{aligned}$$

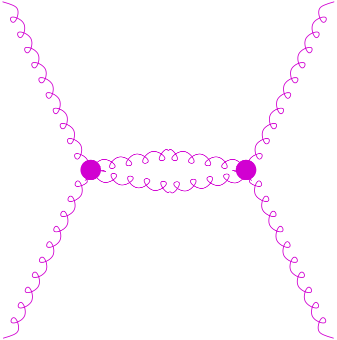
Explicitly

Cut diagram (=inelastic)



Contribution *G*

Uncut (=elastic)



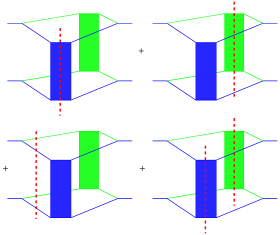
Contribution -*G*

Negative contribution means shadowing / screening

Consider two Pomerons

Total cross section contribution
(at least one +G) **proportional to**

$$\mathbf{0 \times (-G)^2 + 2G(-G) + G^2}$$



Di-jet cross section σ_{dijet}: Each cut Pomeron produces one di-jet =>

$$\sigma_{\mathrm{dijet}} = 0 \times (-G)^2 + 1 \times 2G(-G) + 2 \times G^2$$
$$= \quad 0 \quad - \quad 2G^2 \quad + \quad 2G^2 \quad = \quad 0$$

The different contributions cancel!!

Consider three Pomerons

Total cross section contribution (at least one +G) **proportional to**

$$0 \times (-G)^3 + 3G(-G)^2 + 3G^2(-G) + G^3$$

For di-jet cross section σ_{dijet}, add coefficients (number of di-jets):

$$0 \times (-G)^3 + 1 \times 3G(-G)^2 + 2 \times 3G^2(-G) + 3 \times G^3$$
$$= \quad 0 + 3G^3 - 6G^3 + 3G^3 = \quad 0$$

Again the different contributions cancel!!

Contribution for n Pomerons (k refers to the cut Pomerons):

$$\sigma_{\mathrm{dijet}}^{(n)} \propto \sum_{k=0}^{n} k\, G^{k}\, (-G)^{n-k} \begin{pmatrix} n \\ k \end{pmatrix}$$

$$\propto \sum_{k=0}^{n} (-1)^{n-k}\, k \times \begin{pmatrix} n \\ k \end{pmatrix}$$

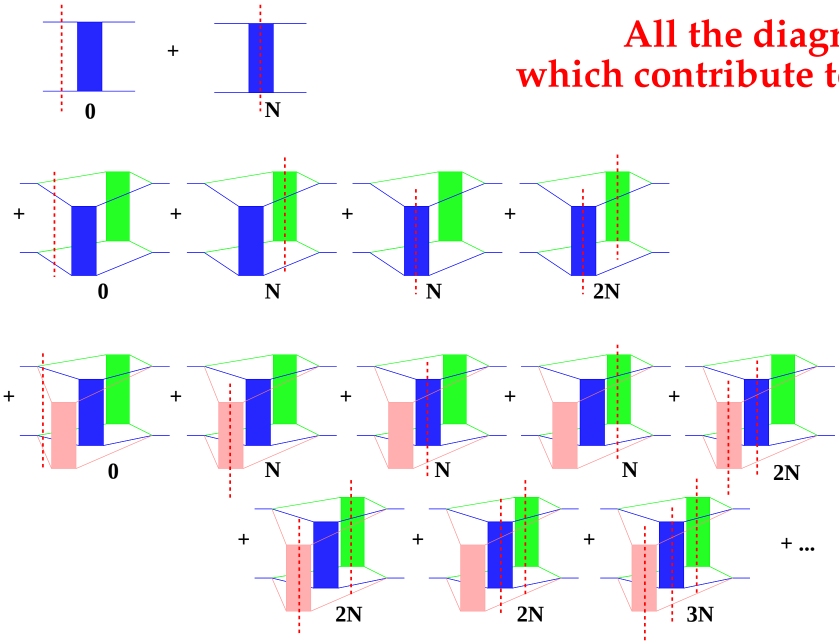
$$= 0 \quad \text{for any } n > 1$$

- **Almost all of the diagrams (i.e. n=2, n=3, ...) do not contribute at all to the inclusive cross section**

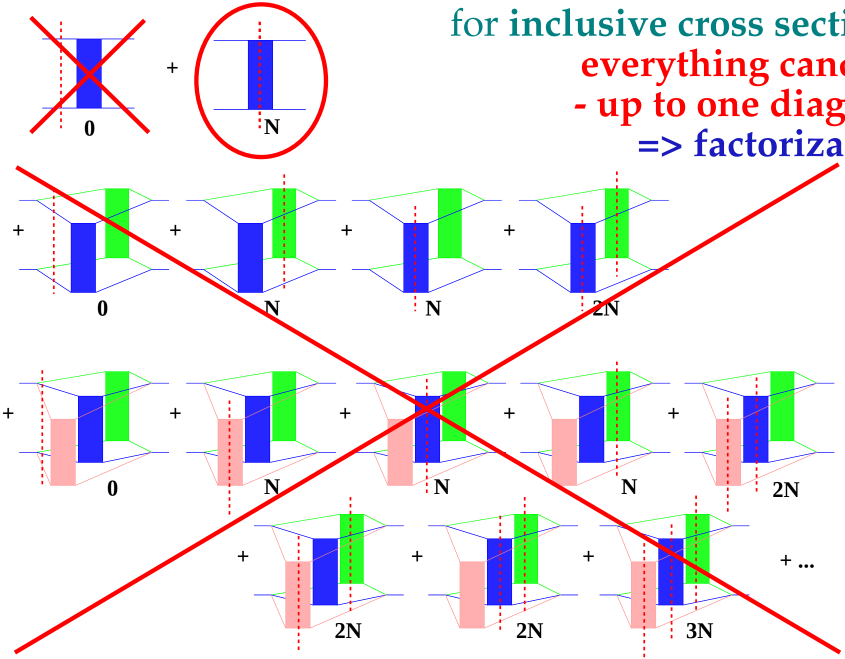
- **Enormous amount of cancellations (interference), only n=1 contributes**

- **AGK cancellations**
  (Abramovskii, Gribov and Kancheli cancellation (1973))

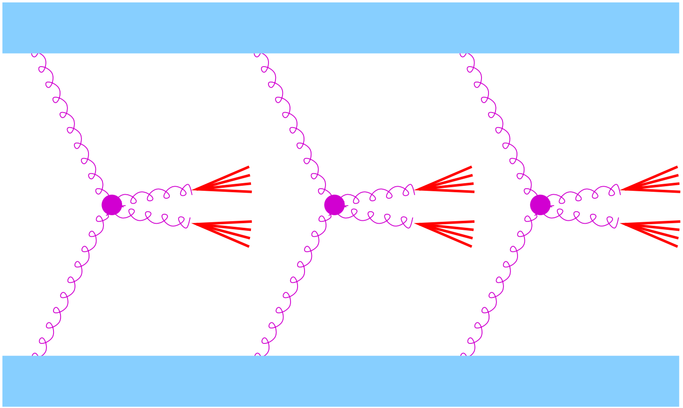
All the diagrams which contribute to pp



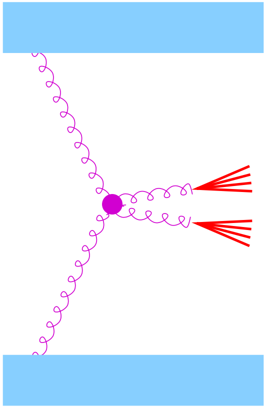
for **inclusive cross sections**
everything cancels
- up to one diagram
=> factorization



Even though the real events show multiple Pomerons



For inclusive cross sections (and only then) a simple diagram is enough



which corresponds to factorization:

$$\sigma_{\mathbf{incl}} = f \otimes \sigma_{\mathbf{elem}} \otimes f$$

Remark:

- □ **We get perfect AGK cancellations in our simplified GR picture (no energy sharing)**

- □ **In the full scheme, it works at large pt (in EPOS4)**

Beyond factorization

Factorization simplifies things enormously!

Extremely useful when computing inclusive di-jet cross sections to study the underlying elementary QCD processes. The full event structure is not needed.

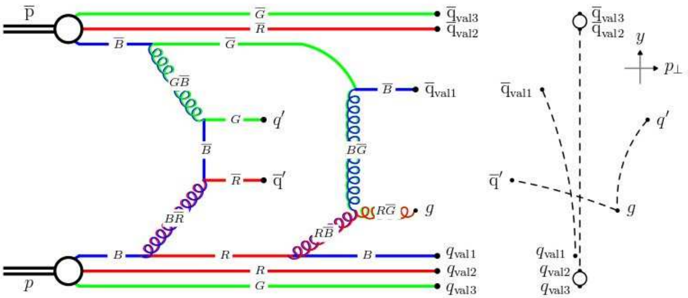
However, many observables require "full events", like everything related to given multiplicity selections.

Two strategies to deal with.

Strategy 1

Start out from factorization, sampling several di-jets from a single diagram,

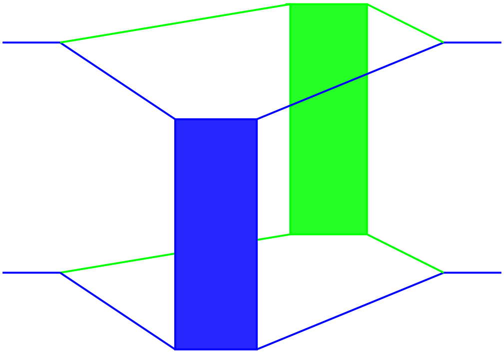
and then attribute them to different subprocesses, redefine color structures (Pythia, Herwig,...)



Strategy 2

Start out from multi-Pomeron S-matrix, sample multi-Pomeron configurations using cutting rule techniques, employing Markov chains

and sample di-jets for each Pomeron, one per Pomeron (EPOS)

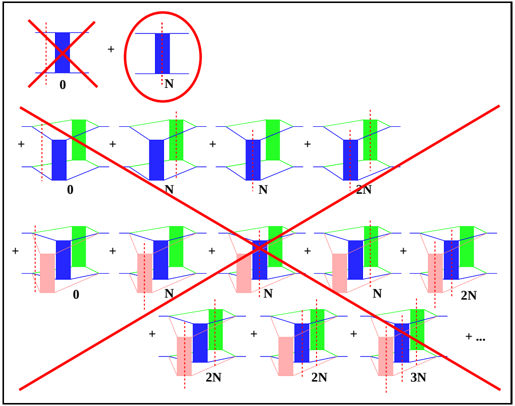


Pros and cons

Strategy	Pros	Cons
Method 1 (PYTHIA)	Simple to realise ———————————— Best method for inclusive cross sections	"Reconstruction" of multiple scattering without solid theoretical basis ———————————— probably not working for small pt ———————————— No obvious extension towards AA
Method 2 (EPOS)	Solid theoretical basis concerning multiple parallel scattering ———————————— Straightforward generalization for AA	Realisation technically demanding ———————————— Factorization not for free, big effort needed to realize the cancellations

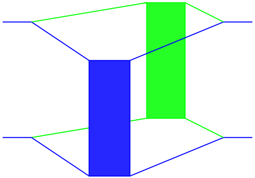
Main problem for the EPOS method:

Since all diagrams are considered:



In case of inclusice cross sections, the corresponding diagrams must actually cancel, which requires high precision and good strategies

AA collisions

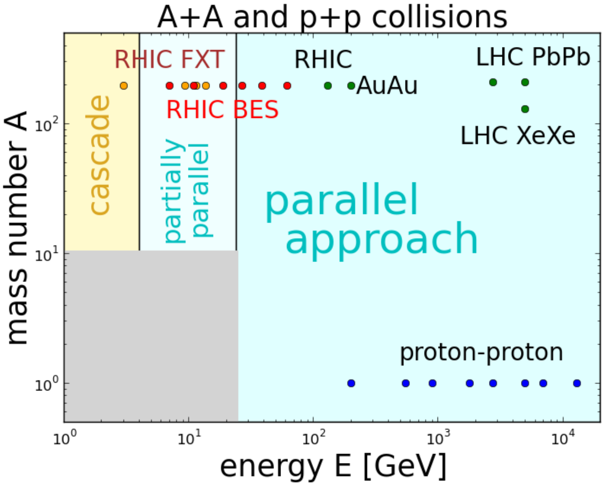


Almost trivial to extend the multiple Pomeron picture to AA.
The T-matrix is essentially a product of the pp expressions:

$$-i \prod_{\text{pairs}} \{iT_{\text{Pom}} \times ... \times iT_{\text{Pom}}\}$$

Again, the difficulty is the fact that realizing AGK cancellations requires big efforts

Crucial! Amounts to binary scaling



So again, the multiple Pomeron approach is difficult (high precision and sophisicated strategies needed to get cancellations)

but there is no real alternative, we need a "parallel approach"

1.7 Glauber and Gribov Regge

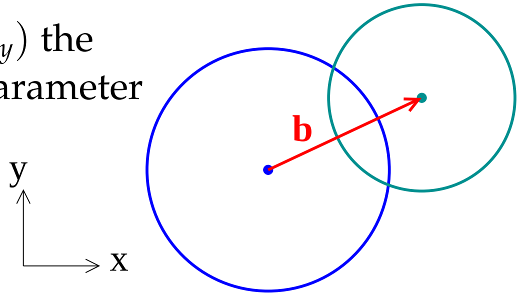
Glauber approach (essentially geometry)
Nucleus-nucleus collision A + B :

- Sequence of independent binary nucleon-nucleon collisions
- Nucleons travel on straight-line trajectories
- The inelastic nucleon-nucleon cross-section σ_{NN} is independent of the number od NN collisions

Monte Carlo version: Two nucleons collide if their transverse distance is less than $\sqrt{\sigma_{NN}/\pi}$.

Analytical formulas for A + B scattering:

- Be ρ_A and ρ_B the (normalized nuclear densities), and
- $b = (b_x, b_y)$ the impact parameter

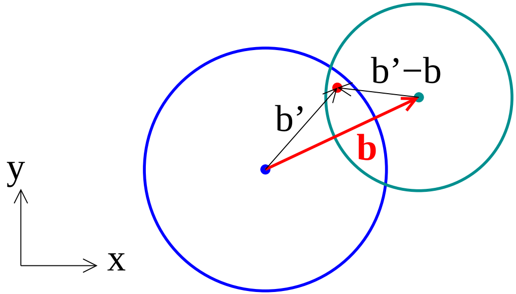


Define integral over nuclear density for each nucleus:

$$T_{A/B}(b') = \int \rho_{A/B}(b', z) dz,$$

and the "thickness function"

$$T_{AB}(b) = \int T_A(b')T_B(b' - b)d^2b'$$



Probability of interaction

(for ρ_A and ρ_B normalized to 1)

$$P = T_{AB}(b)\,\sigma_{NN}$$

Having AB possible pairs: probability of n interactions :

$$P_n = \binom{AB}{n} P^n (1-P)^{AB-n}$$

Probability of at least one interaction (given b):

$$\sum_{n=1}^{AB} P_n = 1 - P_0 = 1 - (1-P)^{AB}$$

And correspondingly the AB cross section :

$$\sigma^{AB} = \int \left\{1 - (1-P)^{AB}\right\} d^2b.$$

(called optical limit).

Probability of an interaction explicitely:

$$\frac{d\sigma^{AB}}{d^2b} = 1 - \{(1 - T_{AB}(b)\,\sigma_{NN})^{AB}\}.$$

Glauber MC formula (with $\sigma_{NN} = \int f(b)d^2b$):

$$\frac{d\sigma^{AB}}{d^2b} = 1 - \left\{\int \prod_{i=1}^{A} d^2b_i^A\, T_A(b_i^A) \prod_{j=1}^{B} d^2b_j^B\, T_B(b_j^B) \prod_{k=1}^{AB} (1 - f)\right\}.$$

In the MC version, one extracts N_{coll}, N_{particip}, and one usually employs a "wounded nucleon approach"

Does this make sense?

Theoretical justification?

... based on relativistic quantum mechanical scattering theory, compatible with QCD

=> Gribov-Regge approach

Gribov Regge for pp, no energy sharing

In the GR framework, we obtain
(neglecting energy sharing)

$$\frac{d\sigma^{pp}}{d^2b} = \sum_{m>0} \sum_{l} \frac{G(b)^m}{m!} \frac{\{-G(b)\}^l}{l!}$$

$$= \sum_{m>0} \frac{G(b)^m}{m!} e^{-G(b)} = \sum_{m} \frac{G(b)^m}{m!} e^{-G(b)} - e^{-G(b)}$$

So

$$\frac{d\sigma^{pp}}{d^2b} = 1 - e^{-G(b)} = f(b)$$

with $f(b)$ being the probability of an interaction at given b.

Gribov Regge for A+B scattering

In the GR framework, defining

$$\int dT_{AB} := \int \prod_{i=1}^{A} d^2b_i^A \, T_A(b_i^A) \prod_{j=1}^{B} d^2b_j^B \, T_B(b_j^B),$$

we obtain (neglecting energy sharing):

$$\frac{d\sigma^{AB}}{d^2b} = \int dT_{AB} \underbrace{\sum_{m_1} ... \sum_{m_{AB}}}_{\sum m_i \neq 0} \prod_{k=1}^{AB} \frac{G(b_k)^{m_k}}{m_k!} e^{-G(b_k)}$$

$$\frac{d\sigma^{AB}}{d^2b} = \int dT_{AB} \underbrace{\sum_{m_1} \ldots \sum_{m_{AB}}}_{\sum m_i \neq 0} \prod_{k=1}^{AB} \frac{G(b_k)^{m_k}}{m_k!} e^{-G(b_k)}$$

$$= \int dT_{AB} \sum_{m_1} \ldots \sum_{m_{AB}} \prod_{k=1}^{AB} \frac{G(b_k)^{m_k}}{m_k!} e^{-G(b_k)} - \prod_{k=1}^{AB} e^{-G(b_k)}$$

$$= \int dT_{AB} \prod_{k=1}^{AB} \underbrace{\sum_{m_k} \frac{G(b_k)^{m_k}}{m_k!}}_{\exp(G(b_k))} e^{-G(b_k)} - \prod_{k=1}^{AB} e^{-G(b_k)}$$

So

$$\frac{\sigma^{AB}}{d^2b} = 1 - \int dT_{AB} \left\{ \prod_{k=1}^{AB} e^{-G(b_k)} \right\}$$

With $f = 1 - e^{-G(b)}$ being the probability of an interaction in pp (with $\sigma^{pp} = \int f(b) d^2b$),

we get the Gribov-Regge result

$$\frac{\sigma^{AB}}{d^2b} = 1 - \left\{ \int dT_{AB} \prod_{k=1}^{AB} (1 - f) \right\}$$

which corresponds to “Glauber Monte Carlo”.

So we find:

In the GR framework (based on quantum mechanics!) we obtain cross section results

- ☐ **corresponding to a simple geometrical picture**
- ☐ **as realized in the Glauber approch**

So we find:

In the GR framework (based on quantum mechanics!) we obtain cross section results

- ☐ **corresponding to a simple geometrical picture**
- ☐ **as realized in the Glauber approch**

BUT ...

... this concern total cross sections!!

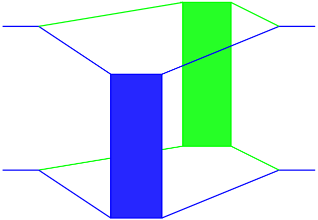
and not at all particle production cross sections

- **In Glauber**
  - **one has (usually) a hard component ($\sim N_{\mathrm{coll}}$)**
  - **and a soft one ($\sim N_{\mathrm{part}}$, wounded nucleons)**
- **In GR** (EPOS)
  - **remnants contribute only at large rapidities,**
  - **otherwise everything is coming from "cut Pomerons" associated to NN scatterings, and one has to account for "shadowing/saturation"**

2 Gribov-Regge & Partons (GRP)

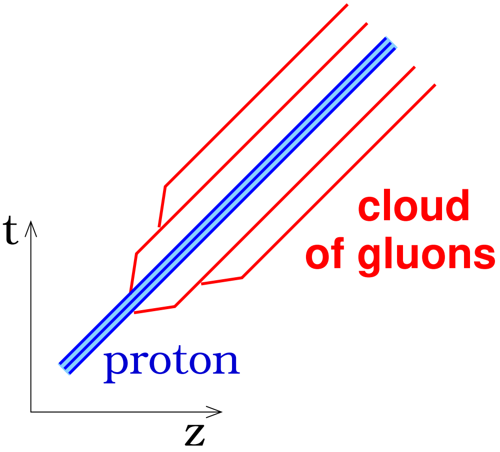
Back to the GR approach employed in EPOS to account for multiple parallel interactions, via the (elastic scattering) **T-matrix**

$$-i\,\{iT_{\mathrm{Pom}} \times ... \times iT_{\mathrm{Pom}}\}$$



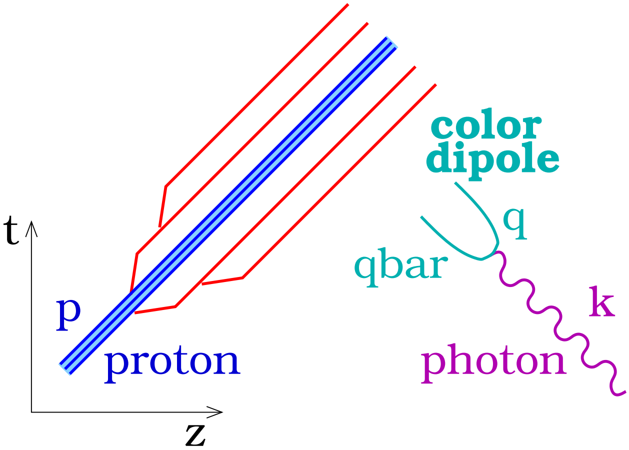
The QCD part is hidden in the "boxes", so what precisely should be put there?

2.1 A fast moving proton



emits successively partons (mainly gluons), quasi-real (large gamma factors)

... which can be probed by a virtual photon
(emitted from an electron)



photon splits into q-qbar

→ Color dipole

p and k are proton and photon momentum

What precisely the photon "sees" depends on two kinematic variables,

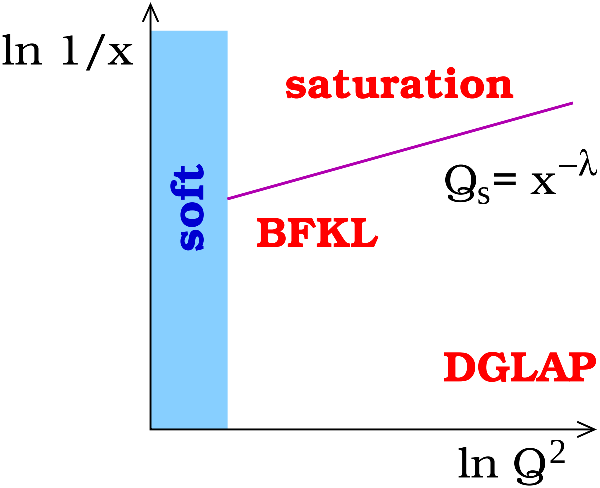
the **virtuality**

$$Q^2 = -k^2$$

and the **Bjorken variable**

$$x = \frac{Q^2}{2pk}$$

which probes partons with momentum fraction x.
It determines also the **approximation scheme** to compute the parton cloud.



DGLAP: summing to all orders of $\alpha_s \ln Q^2$

BFKL: summing to all orders of $\alpha_s \ln \frac{1}{x}$

Linear equations

BFKL (Balitsky, Fadin, Kuraev, and Lipatov):

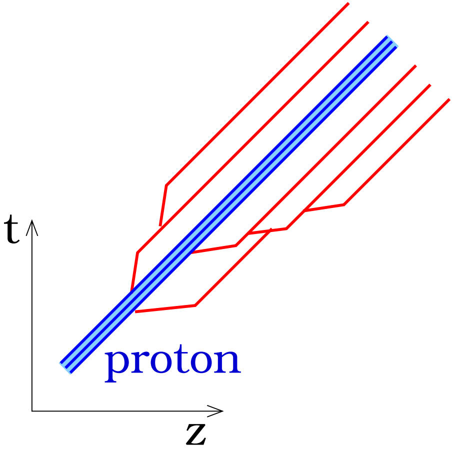
$$\frac{\partial \varphi(x,\boldsymbol{q})}{\partial \ln \frac{1}{x}} \; \frac{\alpha_s N_c}{\pi^2} \int d^2k\, K(\boldsymbol{q},\boldsymbol{k})\varphi(x,\boldsymbol{k})$$

$$\text{with } xg(x,Q^2) = \int_0^{Q^2} \frac{d^2k}{k^2}\varphi(x,\boldsymbol{k}),$$

DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli and Parisi):

$$\frac{\partial g(x,Q^2)}{\partial \ln q^2} = \int_x^1 \frac{dz}{z}\,\frac{\alpha_s}{2\pi}P(z)g(\frac{x}{z},Q^2)$$

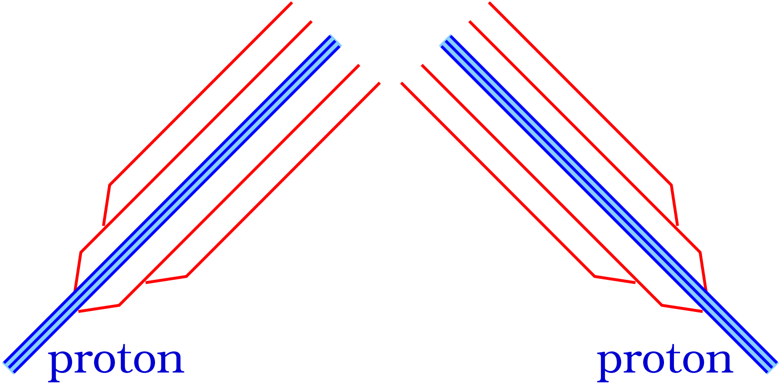
Very large $\ln 1/x$: Saturation domain



Non-linear effects

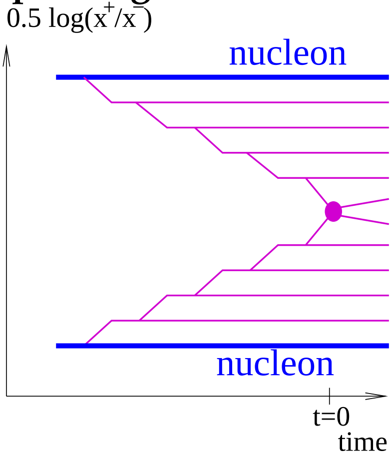
Gluon from one cascade is absorbed by another one

2.2 pp scattering (linear domain)



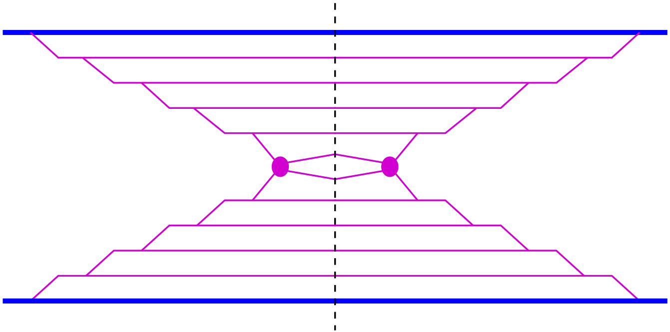
Same evolution as in proton-photon (causality)

Different way of plotting the same reaction



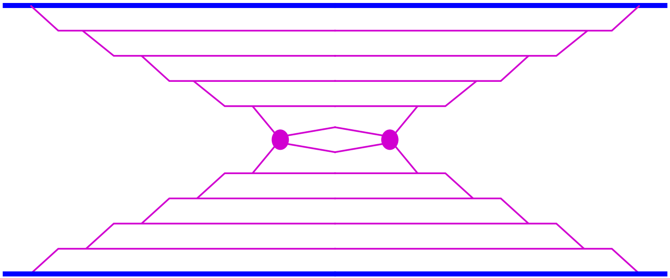
inelastic scattering diagram

Corresponding cut diagram



referred to as “cut parton ladder”
= amplitude squared of the inelastic diagram

Corresponding elastic diagram



referred to as “(uncut) parton ladder”

2.3 Soft domain

Very small $\ln Q^2$: No perturbative treatment!

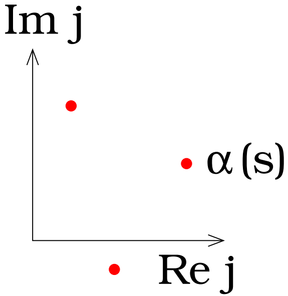
But one may use again the hypothesis of **Lorentz invariance** and **analyticity** of the T-matrix. One starts with a partial wave expansion of the T-matrix (Watson-Sommerfeld transform) :

$$T(t,s) = \sum_{j=0}^{\infty}(2j+1)\mathcal{T}(j,s)P_j(z)$$

with $t \propto z-1$, $z = \cos\vartheta$, P_j: Legendre polynomials.

With $\alpha(s)$ being the right-most pole of $\mathcal{T}(j,s)$ one gets for $t \to \infty$:

$$T(t,s) \propto t^{\alpha(s)}$$



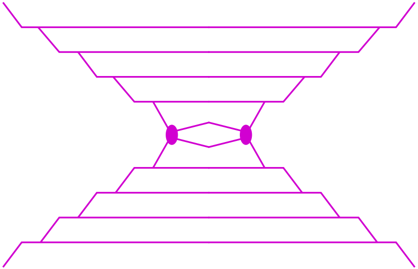
and assuming crossing symmetry one gets the famous asymptotic result

$$T(s,t) \propto s^{\alpha(t)}$$

with the "Regge pole"

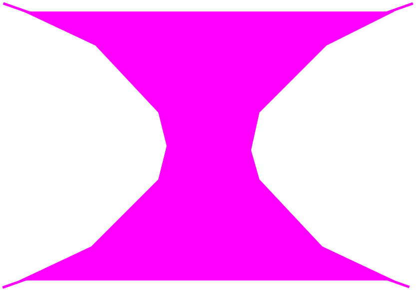
$$\alpha(t) = \alpha(0) + \alpha' t$$

Perturbative:
Parton ladder



T-matrix computed
(DGLAP)

Soft:
Soft Pomeron



gluon fields

T-matrix parametrized

Formulas:

$$T_{\mathrm{soft}}(\hat{s},t) \;=\; 8\pi s_0\, i\, \gamma^2_{\mathrm{Pom-parton}}\, \left(\frac{\hat{s}}{s_0}\right)^{\alpha_{\mathrm{soft}}(0)}$$

$$\times \exp\left(\left\{2R^2_{\mathrm{Pom-parton}} + \alpha'_{\mathrm{soft}} \ln\frac{\hat{s}}{s_0}\right\}\, t\right),$$

Cut soft Pomeron (Schwarz reflection principle):

$$\frac{1}{i}\mathrm{disc}\, T_{\mathrm{soft}}(\hat{s},t)$$

$$= \frac{1}{i}\left[T_{\mathrm{soft}}(\hat{s}+i0,t) - T_{\mathrm{soft}}(\hat{s}-i0,t)\right]$$

$$= 2\mathrm{Im}\, T_{\mathrm{soft}}(\hat{s},t)$$

Interaction cross section,

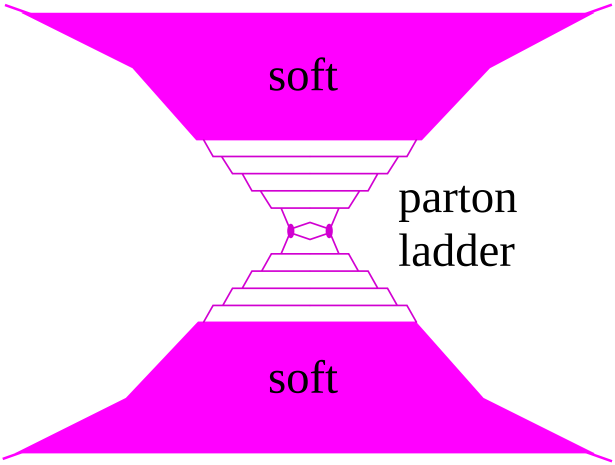
$$\sigma_{\mathrm{soft}}(\hat{s}) = \frac{1}{2\hat{s}} 2\mathrm{Im}\, T_{\mathrm{soft}}(\hat{s},0)\,,$$

$$= 8\pi\gamma_{\mathrm{part}}^{2} \left(\frac{\hat{s}}{s_0}\right)^{\alpha_{\mathrm{soft}}(0)-1},$$

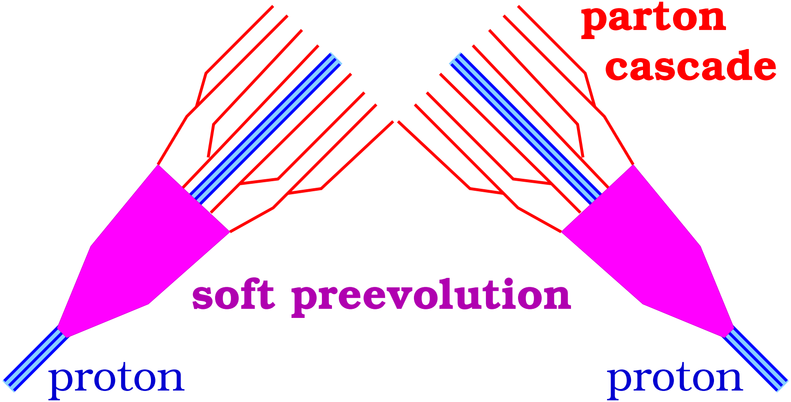
using the optical theorem (with $t = 0$),

which grows faster than data

2.4 Semihard Pomeron



Space-time picture of semihard Pomeron



Hard cross section and amplitude

$$\sigma^{jk}_{\text{hard}}(\hat{s},Q_0^2) = \frac{1}{2\hat{s}} 2\text{Im}\, T^{jk}_{\text{hard}}(\hat{s},t=0)$$
$$= K \sum_{ml} \int dx_B^+ dx_B^- dp_\perp^2 \frac{d\sigma^{ml}_{\text{Born}}}{dp_\perp^2}(x_B^+ x_B^- \hat{s}, p_\perp^2)$$
$$\times E^{jm}_{\text{QCD}}(x_B^+, Q_0^2, M_F^2)\, E^{kl}_{\text{QCD}}(x_B^-, Q_0^2, M_F^2)\theta\left(M_F^2 - Q_0^2\right),$$

One knows (Lipativ, 86): amplitude is imaginary, and nearly independent on t => (with $R^2_{\text{hard}} \simeq 0$) :

$$T^{jk}_{\text{hard}}(\hat{s},t) = i\hat{s}\,\sigma^{jk}_{\text{hard}}(\hat{s},Q_0^2)\;\exp\left(R^2_{\text{hard}}\,t\right)$$

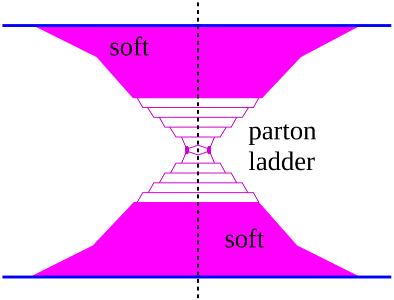
Semihard amplitude :

$$iT_{\mathrm{semihard}}(\hat{s},t)=\sum_{jk}\int_0^1\frac{dz^+}{z^+}\frac{dz^-}{z^-}$$

$$\times\mathrm{Im}\,T^{j}_{\mathrm{soft}}\Big(\frac{s_0}{z^+},t\Big)\;\mathrm{Im}\,T^{k}_{\mathrm{soft}}\Big(\frac{s_0}{z^-},t\Big)\;iT^{jk}_{\mathrm{hard}}(z^+z^-\hat{s},t)$$

(valid for $s\to\infty$ and small parton virtualities except for the ones in the ladder)

Based on these diagrams, one computes T's needed for generating multi-Pomeron configurations,



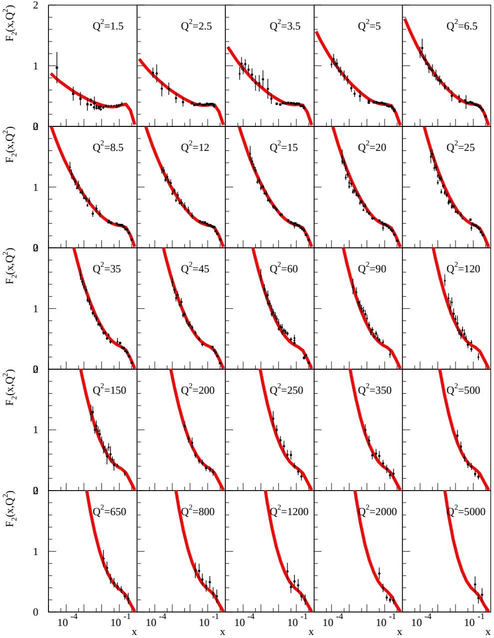
but also computes di-jet cross sections in "factorization mode" as

$$E_3 E_4 \frac{d^6\sigma_{\mathrm{dijet}}}{d^3p_3 d^3p_4} = \sum_{kl} \int\int dx_1 dx_2\, f^k_{\mathrm{PDF}}(x_1,\mu_\mathrm{F}^2) f^l_{\mathrm{PDF}}(x_2,\mu_\mathrm{F}^2)$$

$$\frac{1}{32 s\pi^2} \bar{\sum} |\mathcal{M}^{kl\to mn}|^2 \delta^4(p_1+p_2-p_3-p_4),$$

f_{PDF} are the EPOS PDFs, convolution of soft & DGLAP part

Electron-proton scattering F_2 vs x



To check our f_{PDF}, we can compute

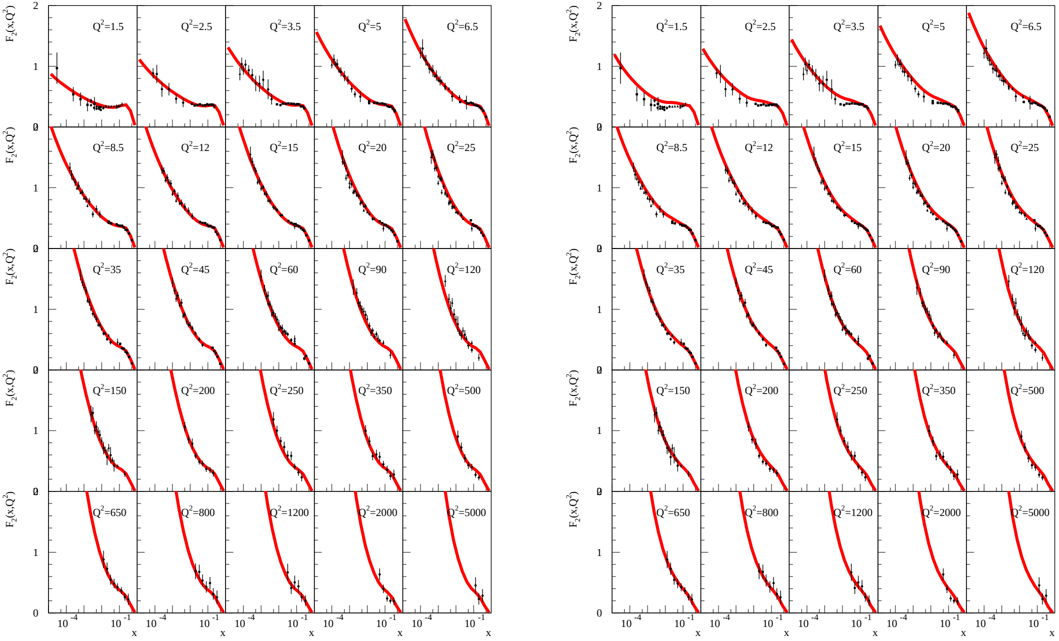
$$F_2 = \sum_k e_k^2\, x\, f_{\mathrm{PDF}}^k(x, Q^2)$$

with

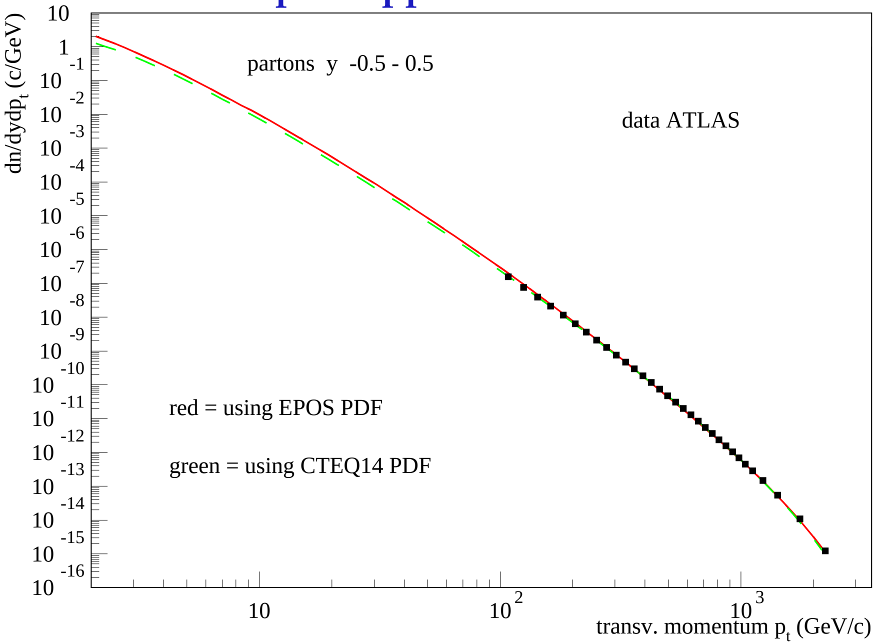
$$x = x_B = \frac{Q^2}{2pq}$$

in the EPOS framework, and compare with data from ZEUS, H1

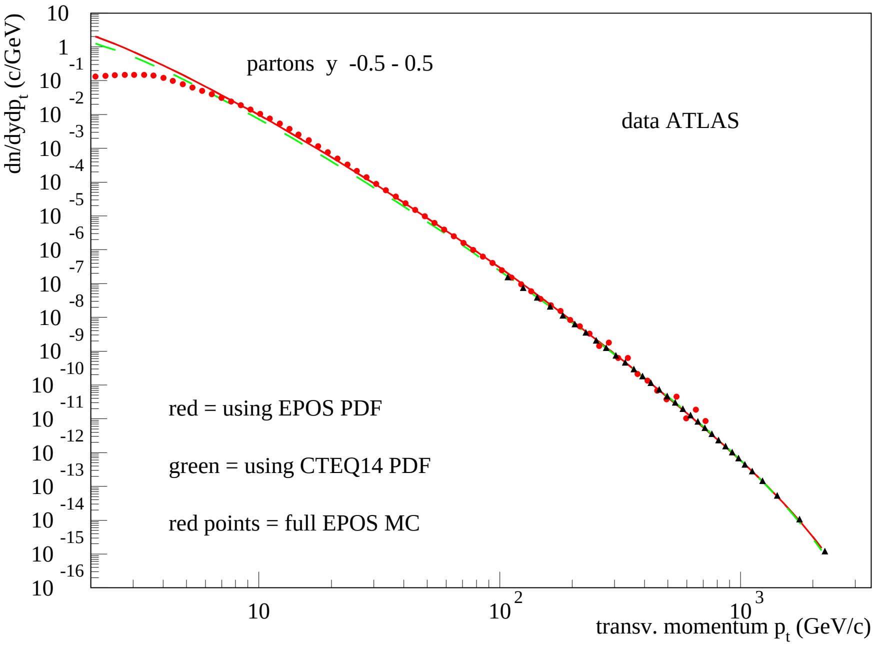
F_2 with EPOS PDF (left) and CTEQ14(5f) PDF (right)



Jet cross section vs pt for pp at 13 TeV



Jet cross section vs pt for pp at 13 TeV



3 Multiple Pomeron exchange in EPOS

The full approach,
going beyond factorization

3.1 Multiple scattering

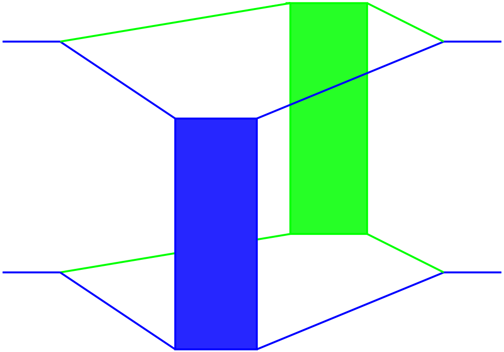
Be T the elastic (pp,pA,AA) scattering T-matrix =>

$$2s\,\sigma_{\mathrm{tot}} = \frac{1}{\mathrm{i}}\mathrm{disc}\,T$$

Basic assumption : Multiple "Pomerons"

$$iT = \sum_k \frac{1}{k!}\left\{iT_{\mathrm{Pom}} \times \ldots \times iT_{\mathrm{Pom}}\right\}$$

Example: 2 “Pomerons”



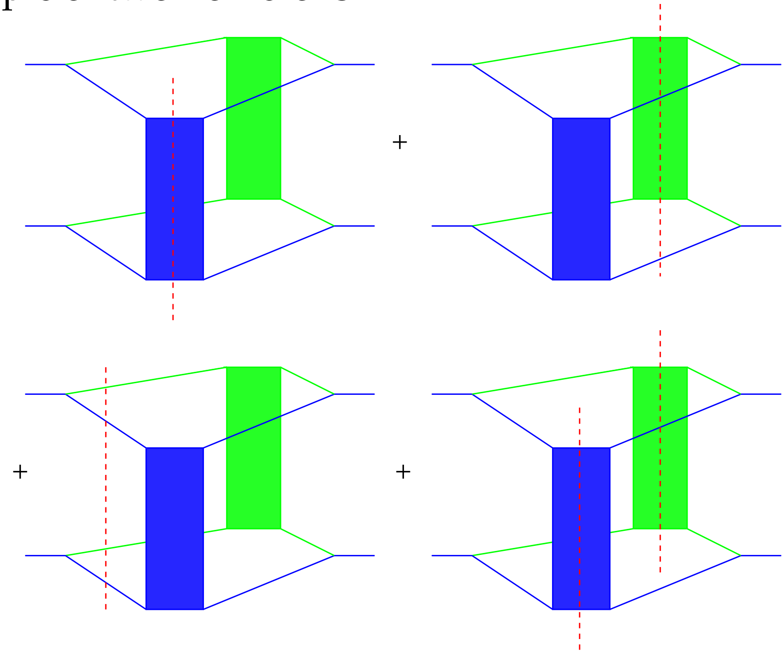
Evaluate

$$\frac{1}{i}\mathrm{disc}\left\{iT_{\mathrm{Pom}}\times ...\times iT_{\mathrm{Pom}}\right\}$$

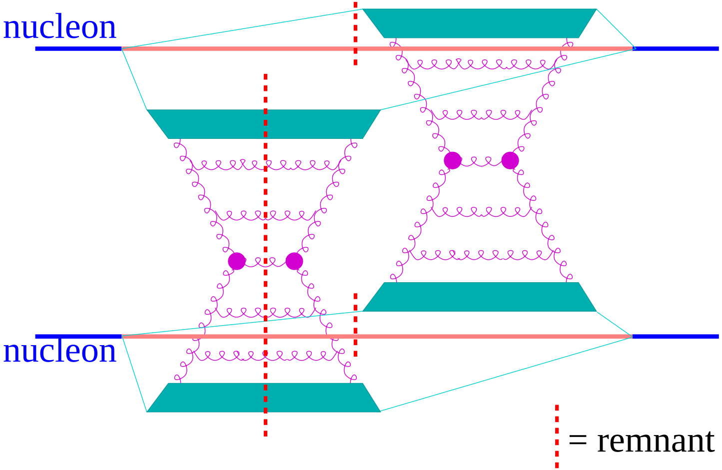
using "cutting rules" :

A "cut" multi-Pomeron diagram
amounts to the sum of all possible cuts

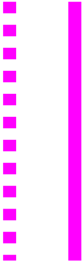
Example of two Pomerons



Using "Pomeron = parton ladder + soft", we have (first diagram)



Using a simplified notation
for "cut" and "uncut" Pomeron

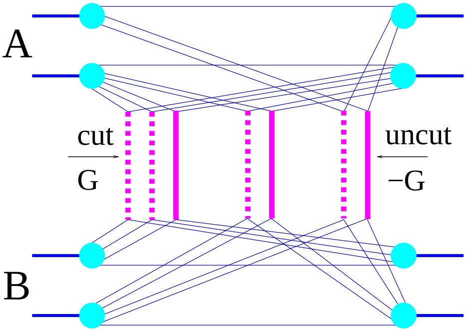


one gets ...

3.2 Complete result

For pp, pA, AA:

$$\sigma^{\mathrm{tot}} = \sum_{\mathrm{cut\,P}} \int \sum_{\mathrm{uncut\,P}} \int$$



partial cross section σ_K

Dotted lines : Cut Pomerons (parton ladders)

$$
\begin{aligned}
\sigma^{\mathrm{tot}} = & \int d^2b \int \prod_{i=1}^{A} d^2b_i^A \, dz_i^A \, \rho_A\Big(\sqrt{(b_i^A)^2 + (z_i^A)^2}\Big) \\
& \prod_{j=1}^{B} d^2b_j^B \, dz_j^B \, \rho_B\Big(\sqrt{(b_j^B)^2 + (z_j^B)^2}\Big) \\
& \sum_{m_1 l_1} \ldots \sum_{m_{AB} l_{AB}} (1 - \delta_{0\Sigma m_k}) \int \prod_{k=1}^{AB} \Bigg(\prod_{\mu=1}^{m_k} dx_{k,\mu}^{+} dx_{k,\mu}^{-} \prod_{\lambda=1}^{l_k} d\tilde{x}_{k,\lambda}^{+} d\tilde{x}_{k,\lambda}^{-} \Bigg) \Bigg\{ \\
& \prod_{k=1}^{AB} \Bigg(\frac{1}{m_k!} \frac{1}{l_k!} \prod_{\mu=1}^{m_k} G(x_{k,\mu}^{+}, x_{k,\mu}^{-}, s, |\vec{b} + \vec{b}_{\pi(k)}^{A} - \vec{b}_{\tau(k)}^{B}|) \\
& \prod_{\lambda=1}^{l_k} -G(\tilde{x}_{k,\lambda}^{+}, \tilde{x}_{k,\lambda}^{-}, s, |\vec{b} + \vec{b}_{\pi(k)}^{A} - \vec{b}_{\tau(k)}^{B}|) \Bigg) \\
& \prod_{i=1}^{A} \Bigg(1 - \sum_{\pi(k)=i} x_{k,\mu,}^{+} - \sum_{\pi(k)=i} \tilde{x}_{k,\lambda}^{+} \Bigg)^{\alpha} \prod_{j=1}^{B} \Bigg(1 - \sum_{\tau(k)=j} x_{k,\mu}^{-} - \sum_{\tau(k)=j} \tilde{x}_{k,\lambda}^{-} \Bigg)^{\alpha} \Bigg\}
\end{aligned}
$$

- **Complicated due to strict energy sharing**

  => 10,000,000-dimensional intergrals, not separable

- **but doable**

  - **Parameterizations for** $G(x^+, x^-, s, b)$
  - **Analytical integrations**
  - **Employing Markov chain techniques**

Step 1:

- □ We compute **partial cross sections** σ_K for particular configurations K via analytical integration

- □ K is a multi-dimensional variable
  for example for double scattering in pp with two Pomerons involved: $K = \{x_1^+, x_1^-, \vec{p}_{t1}, x_2^+, x_2^-, \vec{p}_{t2}\}$

- □ Configurations K in AA scattering may be quite complex

Step 2:

The partial cross sections σ_K can (properly normalized) be

- ☐ interpreted as **probability distributions**,
- ☐ enabling us to use Monte Carlo techniques to **generate configurations *K*** **using Markov chain techniques**

3.3 Configurations via Markov chains

Consider a sequence of multidimensional random numbers (or better random configurations)

$$x_1\,,\; x_2\,,\;\; x_3, ...$$

with f_t being the law for x_t.

A homogeneous Markov chain is defined as

$$f_t(x) = \sum_{x'} f_{t-1}(x')p(x' \to x).$$

with $p(x' \to x)$ being the transition probability (or matrix). Normalization : $\sum_x p(x' \to x) = 1$.

Let f be the law for x_t. The law for x_{t+1} is

$$\sum_a f(a)\, p(a \to b)\,.$$

One defines an operator T (comme Translation)

$$Tf(b) = \sum_a f(a)\, p(a \to b)\,.$$

So Tf is the law for x_{t+1} when f is the law for x_t.

A law is called stationary if $Tf = f$.

> Theorem: If a stationary law $Tf = f$ exists, then $T^k f_1$ converges towards f (which is unique) for any f_1.

So to generate random configurations according to some (given) law f,

- □ one constructs a T such that $Tf = f$
- □ and then considers $f_1 \to Tf_1 \to T^2 f_1$...
- □ and constructs the corresponding random configurations

One needs, for a given law f,
to **find a transition matrix p such that $Tf = f$**

Sufficient condition (detailed balance):

$$f(a)\,p(a \to b) = f(b)\,p(b \to a)\,,$$

Proof :

$$\begin{aligned} Tf(b) &= \sum_a f(a)\,p(a \to b) \\ &= \sum_a f(b)\,p(b \to a) \\ &= f(b) \sum_a p(b \to a) \\ &= f(b)\,. \end{aligned}$$

3.4 Metropolis alorithm

Definitions:

$$p_{ab} = p(a \to b),$$
$$f_a = f(a).$$

Take

$$p_{ab} = w_{ab}\, u_{ab}. \qquad (a \neq b).$$

with

w_{ab} : proposal matrix ($\sum_b w_{ab} = 1$)

u_{ab} : acceptance matrix ($u_{ab} \leq 1$)

This is NOT the simple acceptance-rejection method!!

Detailed balance:

$$f_a\, p_{ab} = f_b\, p_{ba}$$

amounts to

$$f_a\, w_{ab}\, u_{ab} = f_b\, w_{ba}\, u_{ba}\,,$$

or

$$\frac{u_{ab}}{u_{ba}} = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\,.$$

The expression

$$\frac{u_{ab}}{u_{ba}} = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\,.$$

is solved by

$$u_{ab} = F\left(\frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\right),$$

with a function F with

$$\frac{F(z)}{F(\frac{1}{z})} = z\,.$$

$$\mathrm{Proof:}\;\; \mathrm{With}\; z \equiv \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}\,\mathrm{one\,finds}\;:\; \frac{u_{ab}}{u_{ba}} = \frac{F(z)}{F(\frac{1}{z})} = z = \frac{f_b}{f_a}\,\frac{w_{ba}}{w_{ab}}.$$

The F according to Metropolis is

$$F(z) = \min(z, 1)\,.$$

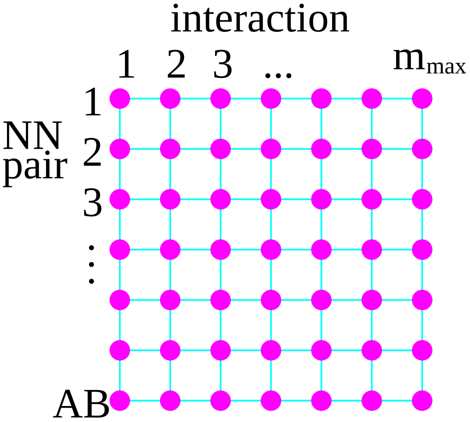
One finds indeed

$$\frac{F(z)}{F(\frac{1}{z})} = \frac{\min(z,1)}{\min(\frac{1}{z},1)} = \left\{ \begin{array}{lll} z/1 & \text{pour} & z \leq 1 \\ 1/\frac{1}{z} & \text{pour} & z > 1 \end{array} \right\} = z\,.$$

So one proposes for each iteration a new configuation b according to some w_{ab}, and accepts it with probability

$$u_{ab} = \min\left(\frac{f_b}{f_a}\frac{w_{ba}}{w_{ab}}, 1\right).$$

Configuration lattice, define w_{ab} such that b changes w.r.t. a only on one lattice site (like Ising model Metropolis)

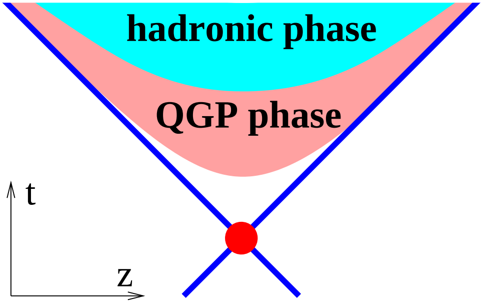


Long iterations, but allows to generate very complex configurations according to very complex laws.

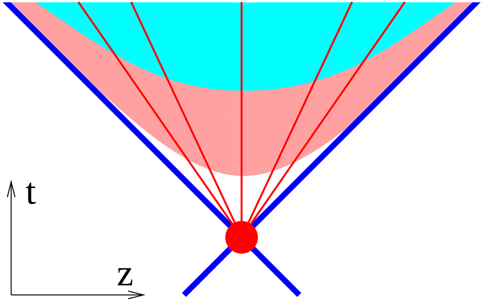
4 Secondary interactions (overview)

4.1 Primary and secondary interactions

So far we discussed primary interactions (the red point)



Milne coordiantes are used to describe evolution



Proper time (hyperbolas)

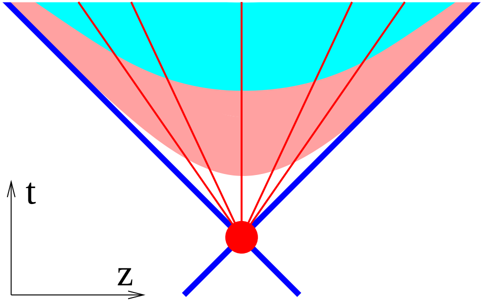
$$\tau = \sqrt{t^2 - z^2}$$

Space-time rapidity
(red lines)

$$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$$

(not pseudorapidity)

Primary interactions determine matter distribution in η_s



and in essentially any scenario η_s correspnds to the average rapidity (of volume cells)

$$< y > \approx \eta_s$$

so primary interactions determine "essentially" the rapidity distrbution

$$\text{with } y = \frac{1}{2} \ln \frac{E + P_z}{E - P_z}$$

Basic structure of EPOS (for modelling pp, pA, AA)

- **Primary interactions**
  Multiple scattering, instantaneously, in parallel
  (Gribov-Regge & Partons, GRP)

- **Secondary interactions**
  formation of "matter" which expands collectively, like a fluid, decays statistically

- **Primary interactions affect very strongly the evolution!**

4.2 Secondary interactions: An example

In this section:

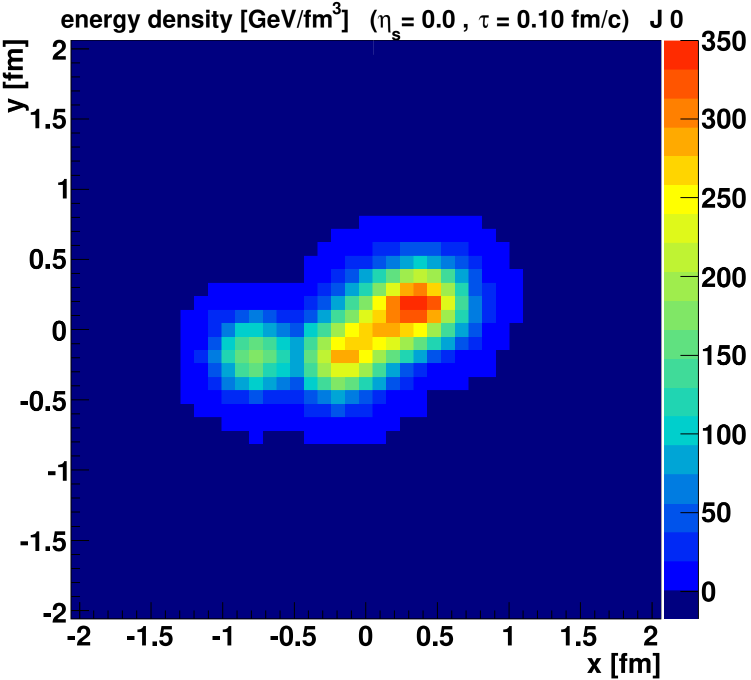
An example of a EPOS simulation

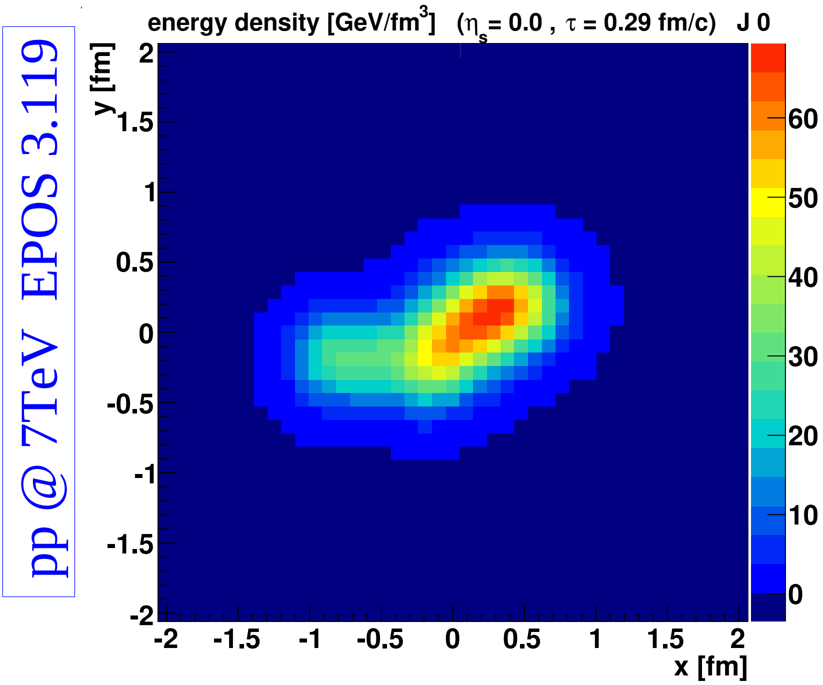
of expanding matter in pp scattering

with initial conditions from GRP

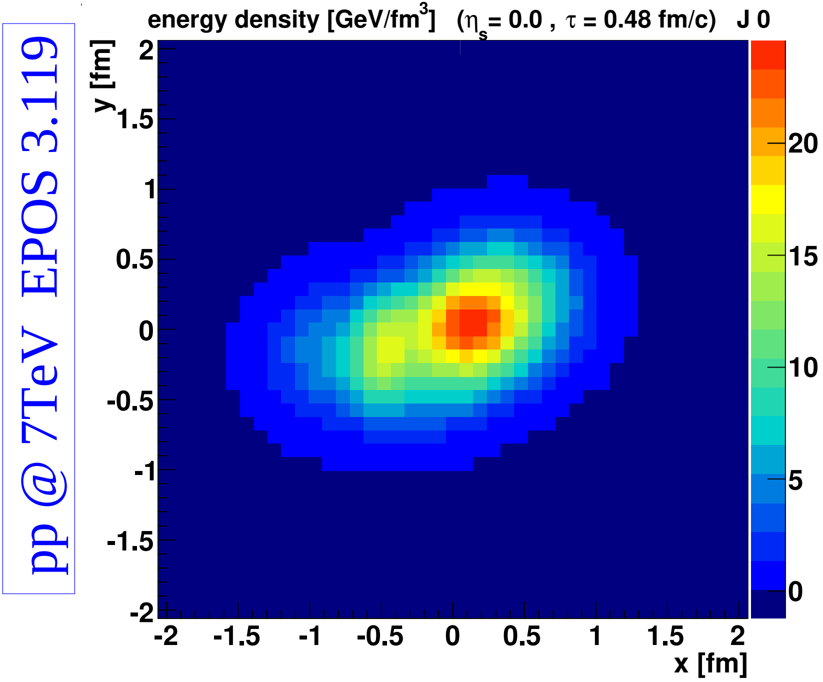
In the following sections: consequences

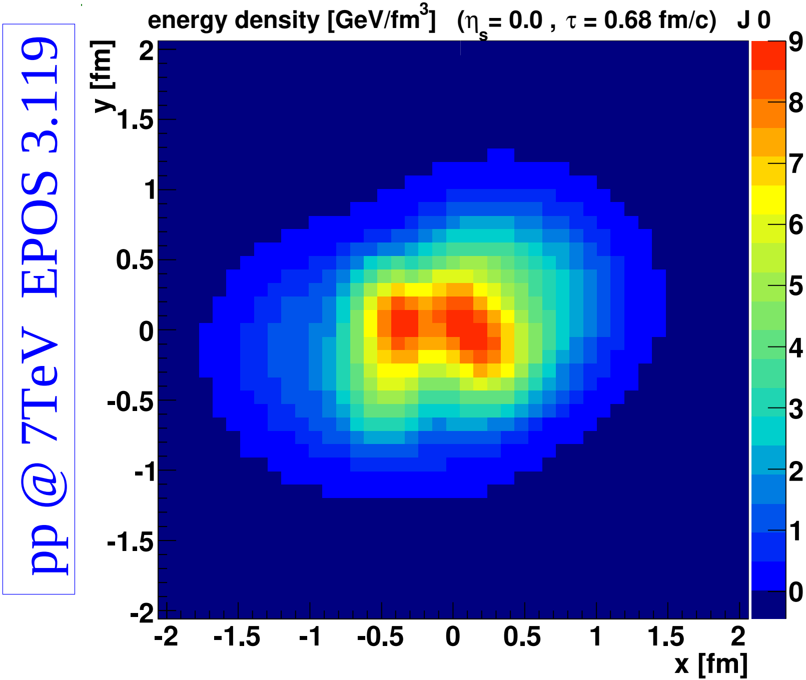
pp @ 7TeV EPOS 3.119

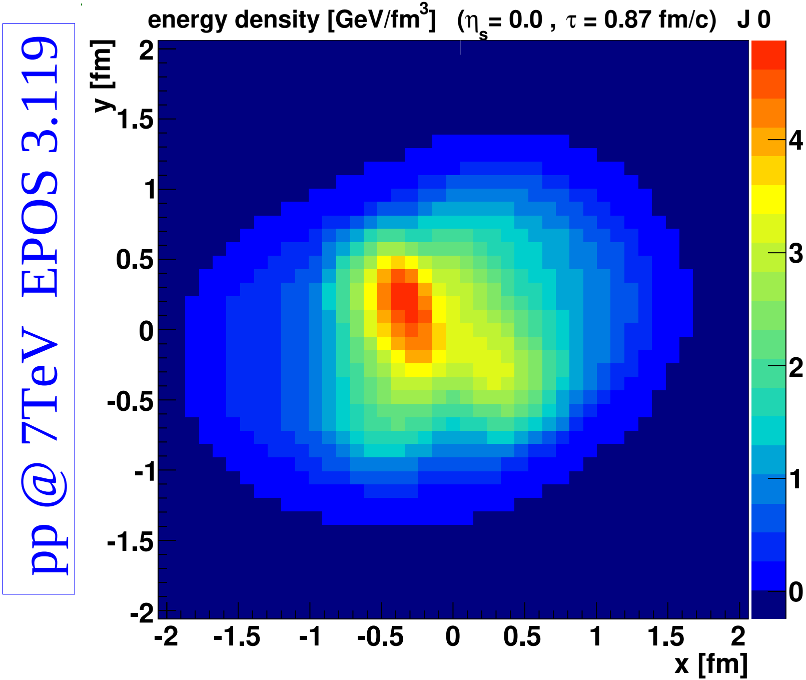




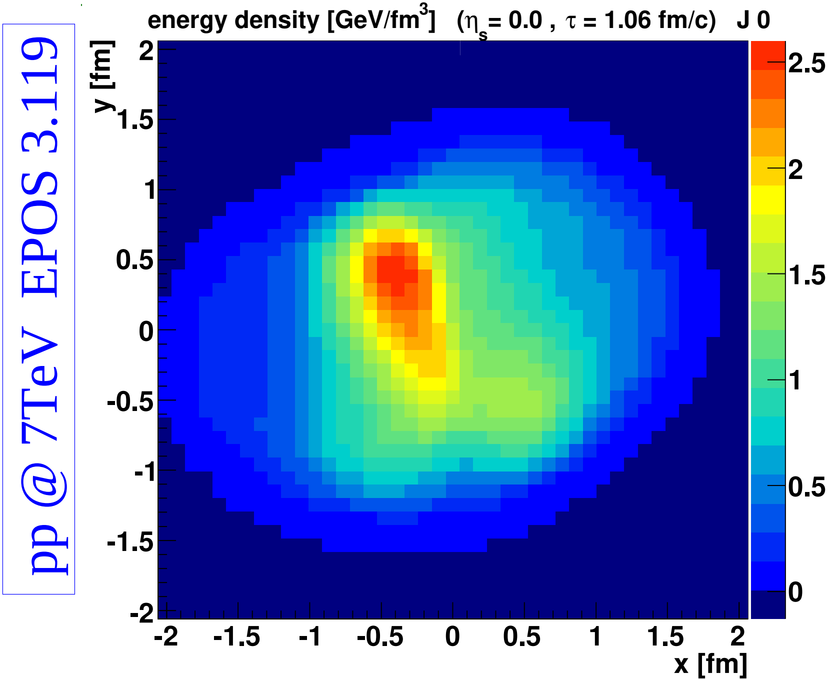
pp @ 7TeV EPOS 3.119
energy density [GeV/fm³] (η_s = 0.0 , τ = 0.29 fm/c) J 0
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
60
50
40
30
20
10
0

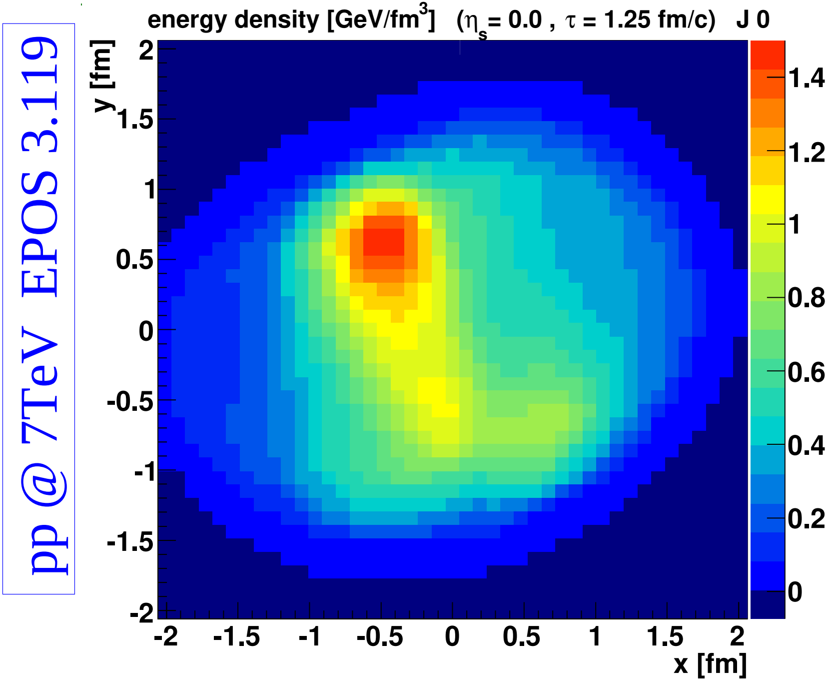

energy density [GeV/fm^3] (η_s = 0.0 , τ = 0.48 fm/c) J 0
pp @ 7TeV EPOS 3.119
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
20
15
10
5
0

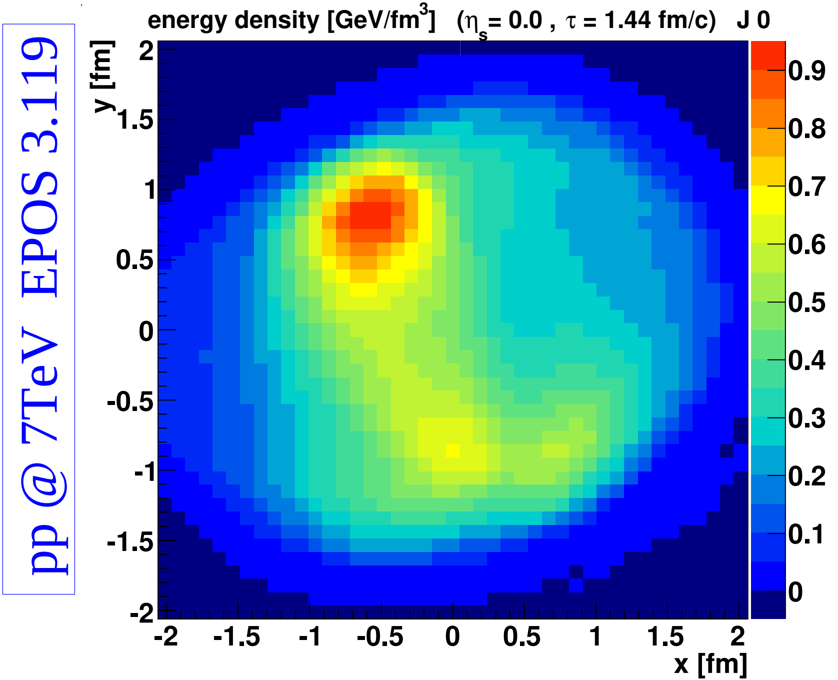

pp @ 7TeV EPOS 3.119
energy density [GeV/fm^3] (η_s = 0.0 , τ = 0.68 fm/c) J 0
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
9
8
7
6
5
4
3
2
1
0

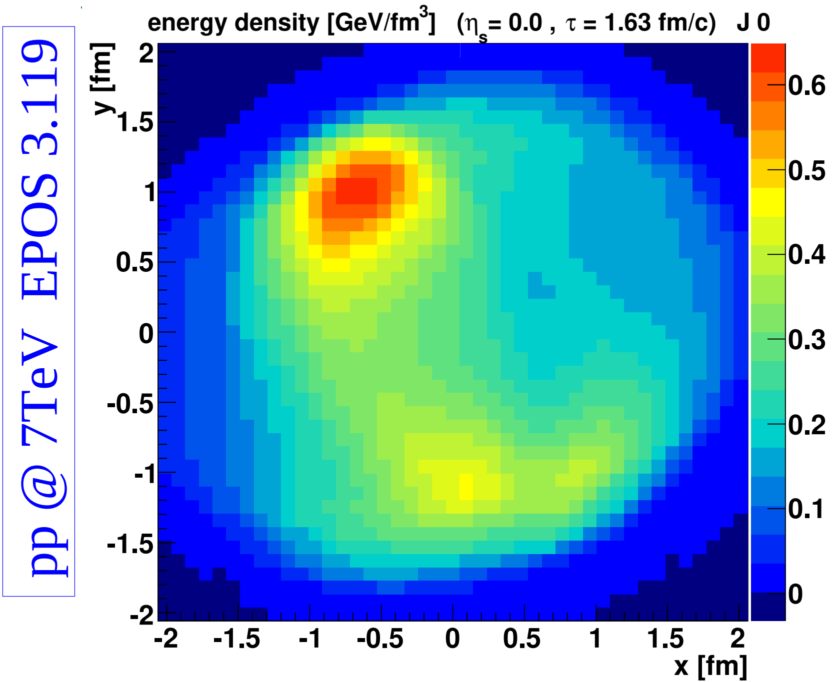


pp @ 7TeV EPOS 3.119
energy density [GeV/fm^3] (η_s = 0.0 , τ = 0.87 fm/c) J 0
y [fm]
x [fm]

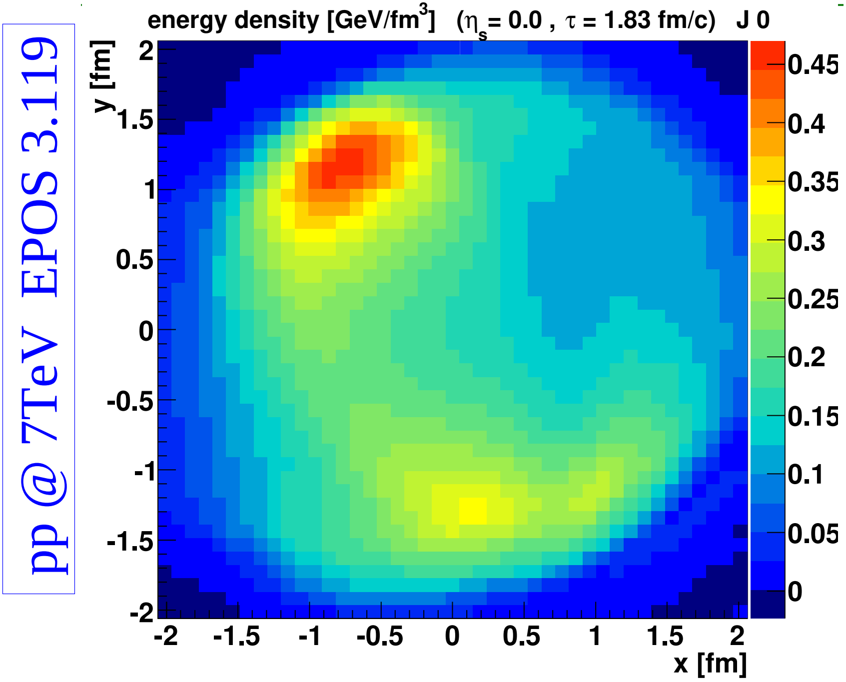

pp @ 7TeV EPOS 3.119
energy density [GeV/fm^3] (η_s = 0.0 , τ = 1.06 fm/c) J 0
y [fm]
x [fm]
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

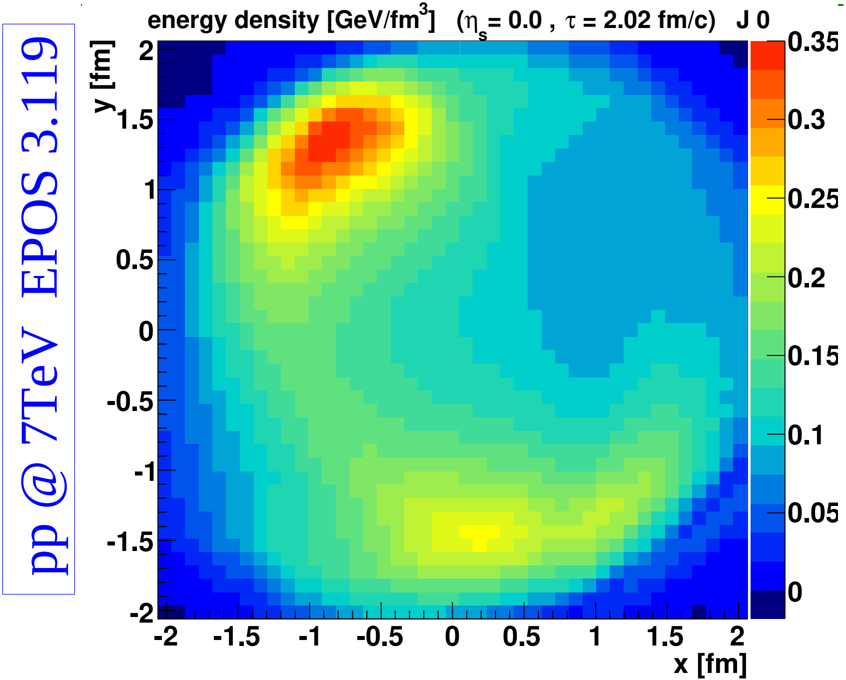

pp @ 7TeV EPOS 3.119
energy density [GeV/fm^3] (η_s = 0.0 , τ = 1.25 fm/c) J 0
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1.4
1.2
1
0.8
0.6
0.4
0.2
0

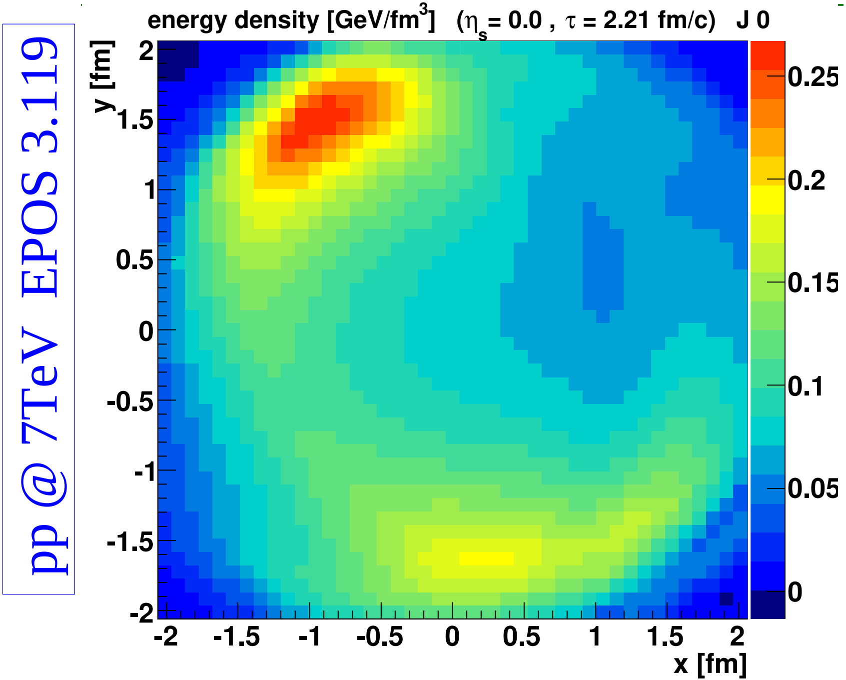

pp @ 7TeV EPOS 3.119
energy density [GeV/fm³] (η_s = 0.0 , τ = 1.44 fm/c) J 0
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

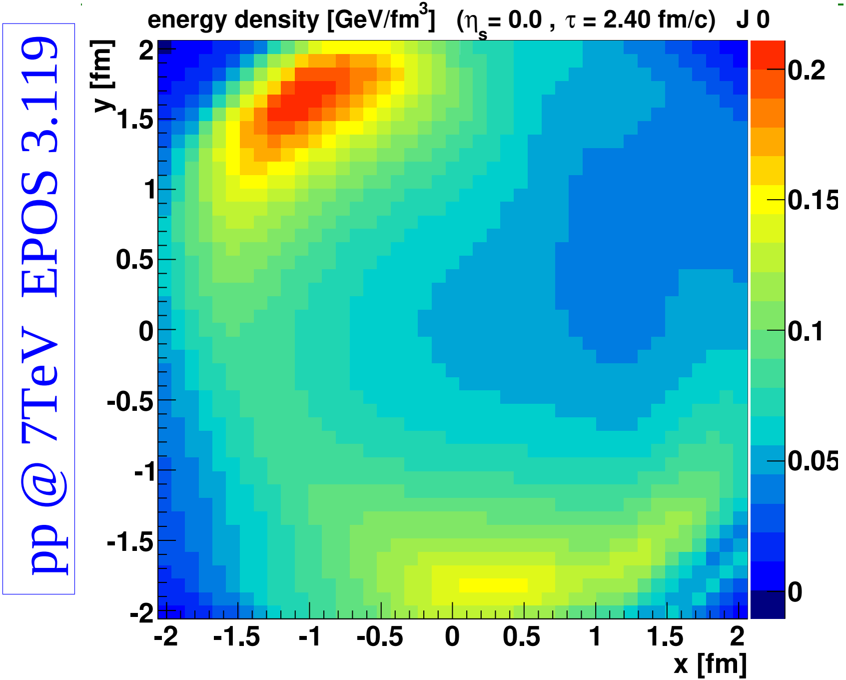


energy density [GeV/fm^3] (η_s = 0.0 , τ = 1.63 fm/c) J 0
pp @ 7TeV EPOS 3.119
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.6
0.5
0.4
0.3
0.2
0.1
0

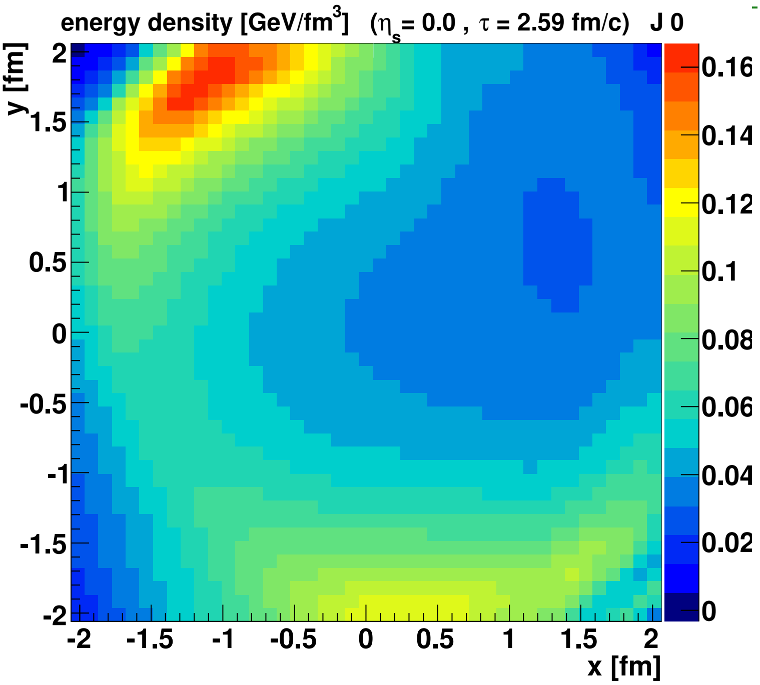

pp @ 7TeV EPOS 3.119
energy density [GeV/fm^3] (η_s = 0.0 , τ = 1.83 fm/c) J 0
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0


pp @ 7TeV EPOS 3.119
energy density [GeV/fm³] (η_s = 0.0 , τ = 2.02 fm/c) J 0
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0


pp @ 7TeV EPOS 3.119
energy density [GeV/fm³] (η_s = 0.0 , τ = 2.21 fm/c) J 0
y [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x [fm]
0.25
0.2
0.15
0.1
0.05
0

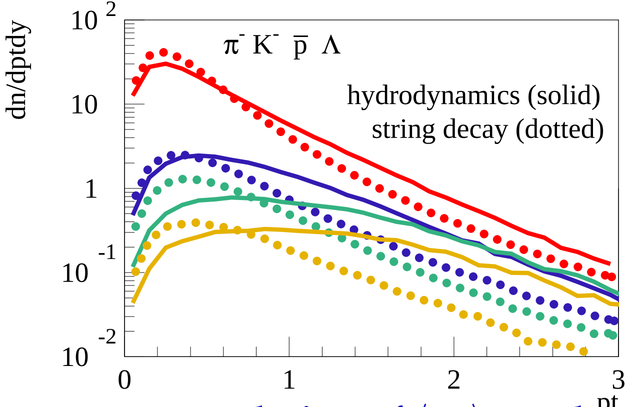

pp @ 7TeV EPOS 3.119
energy density [GeV/fm^3] (η_s = 0.0 , τ = 2.40 fm/c) J 0
y [fm]
x [fm]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.2
0.15
0.1
0.05
0

pp @ 7TeV EPOS 3.119



4.3 Radial flow visible in particle distributions

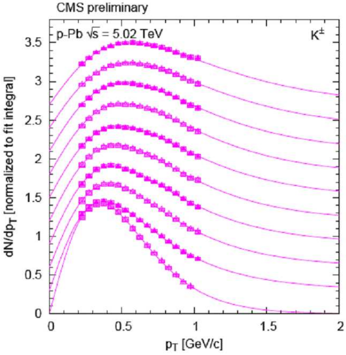
Particle spectra affected by radial flow

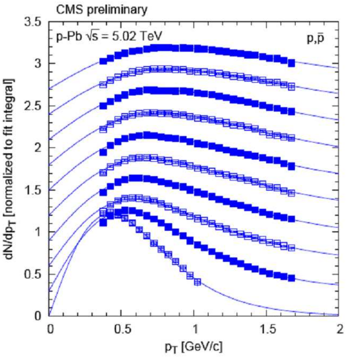


=> mass ordering of $\langle p_t \rangle$, lambda/K increase

pPb at 5TeV

CMS, arXiv:1307.3442



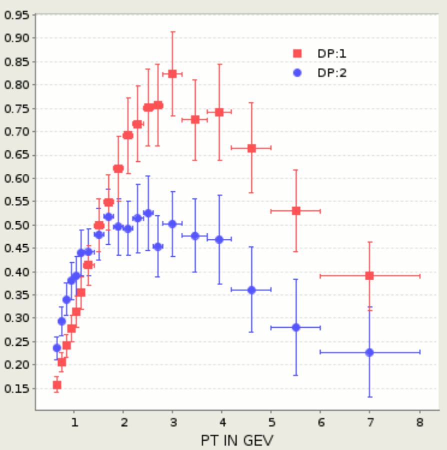


Strong variation of shape with multiplicity

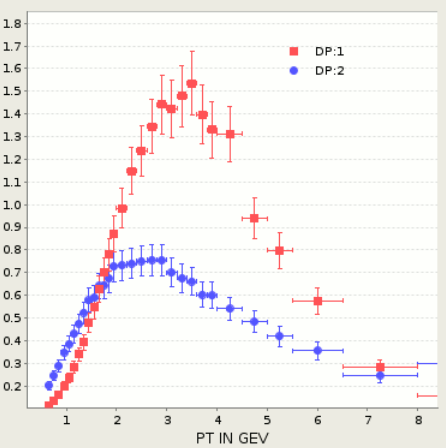
for kaon and even more for proton pt spectra

(flow like)

Λ / K_s versus pT (high compared to low multiplicity) in pPb (left) similar to PbPb (right)



ALICE (**2013**) arXiv:1307.6796



ALICE (**2013**) arXiv:1307.5530
Phys. Rev. Lett. 111, 222301 (2013)

In AA: partially due to flow

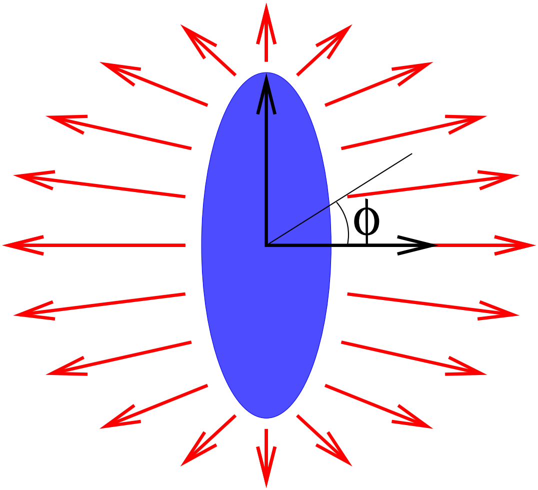
4.4 Ridges & flow harmonics

Anisotropic radial flow
visible in dihadron-correlations

$$R = \frac{1}{N_{\mathrm{trigg}}} \frac{dn}{d\Delta\phi\Delta\eta}$$

Anisotropic flow due to initial azimuthal anisotropies

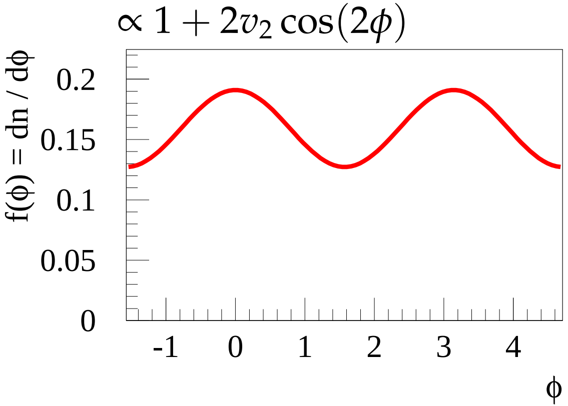
Initial "elliptical" matter distribution:



Preferred expansion along $\phi = 0$ and $\phi = \pi$

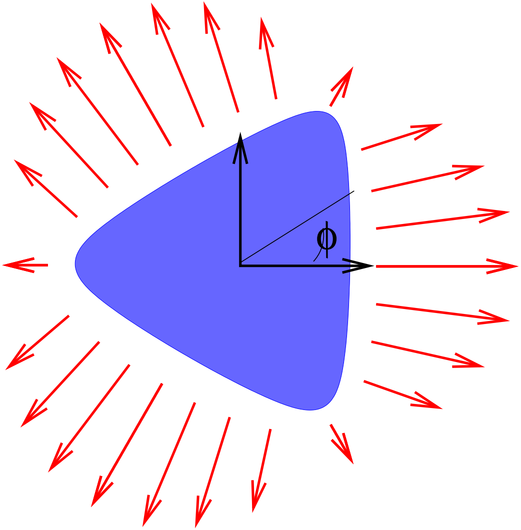
η_s-invariance same form at any η_s
$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$

Particle distribution: Preferred directions $\phi = 0$ and $\phi = \pi$



Dihadrons:
preferred $\Delta\phi = 0$ and $\Delta\phi = \pi$ (even for big $\Delta\eta$)

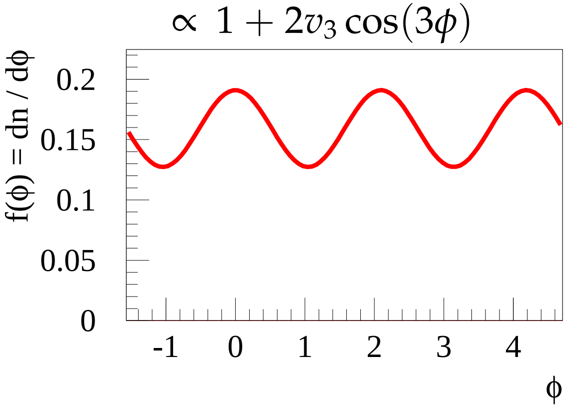
Initial "triangular" matter distribution:



Preferred expansion along $\phi = 0$, $\phi = \frac{2}{3}\pi$, and $\phi = \frac{4}{3}\pi$

η_s-invariance

Particle distribution: Preferred directions $\phi = 0$, $\phi = \frac{2}{3}\pi$, and $\phi = \frac{4}{3}\pi$



Dihadrons:
preferred $\Delta\phi = 0$, and $\Delta\phi = \frac{2}{3}\pi$, and $\Delta\phi = \frac{4}{3}\pi$
(even for large $\Delta\eta$)

In general, superposition of several eccentricities ε_n,

$$\varepsilon_n e^{in\psi_n^{PP}} = -\frac{\int dxdy\, r^2 e^{in\phi} e(x,y)}{\int dxdy\, r^2 e(x,y)}$$

Particle distribution characterized by harmonic flow coefficients

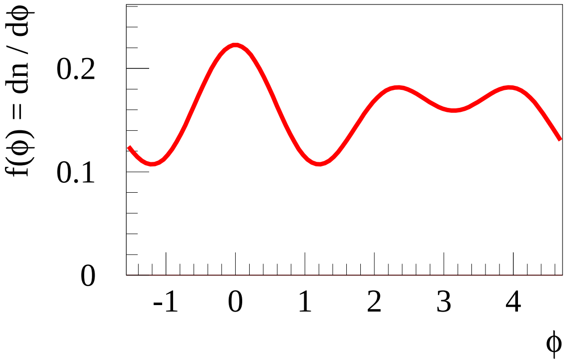
$$v_n e^{in\psi_n^{EP}} = \int d\phi\, e^{in\phi} f(\phi)$$

At $\phi = 0$:
The **ridge**

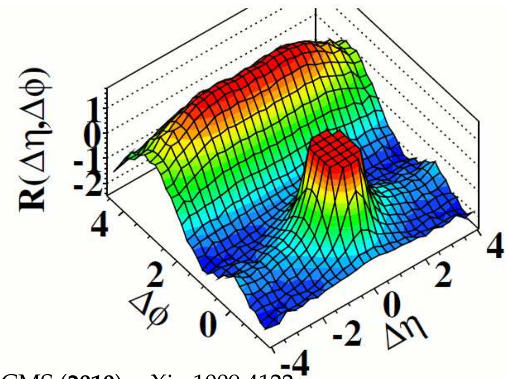
(extended in η)

Awayside peak may originate from jets, not the ridge (for large $\Delta\eta$)

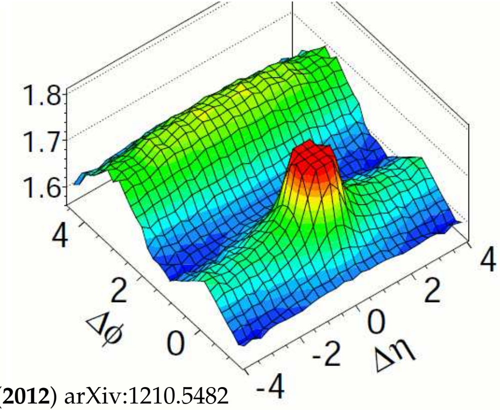
Here, v_2 and v_3 non-zero
$\propto 1 + 2v_2 \cos(2\phi) + 2v_3 \cos(3\phi)$



CMS: Ridges (in dihadron correlation functions) also seen in pp (left) and pPb (right)



CMS (**2010**) arXiv:1009.4122
JHEP 1009:091,2010

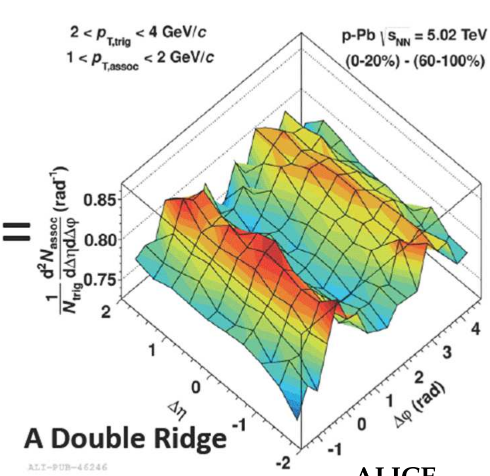


CMS (**2012**) arXiv:1210.5482
Phys. Lett. B 718 (2013) 795

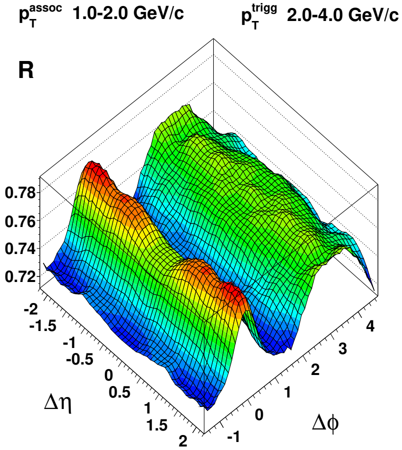
Looks like flow !

Ridges also realized in simulations in pPb (and even pp)

Central - peripheral (to remove jets) Phys. Lett. B 726 (2013) 164-177



ALICE



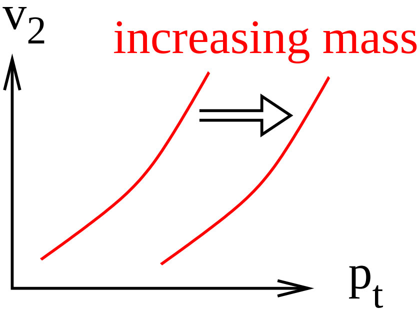
EPOS3.074

4.5 Flow harmonics, identified particles

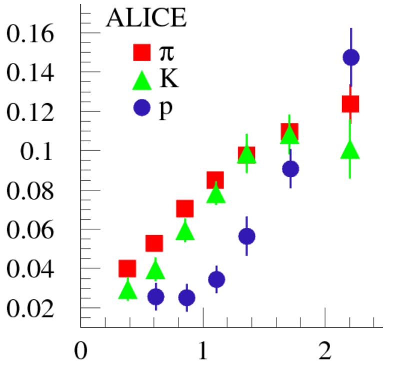
Flow shifts particles to higher p_t

Effect increases with mass

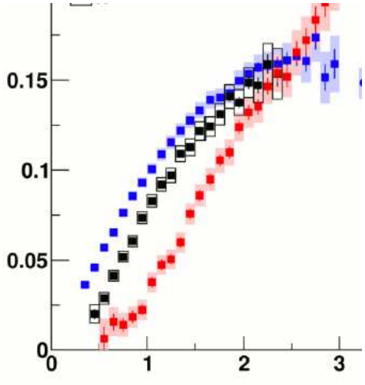
Also true for v_2 vs p_t



ALICE: v2 versus pT: mass splitting (π, K, p) in pPb (left) similar to PbPb (right)



ALICE (**2013**) arXiv:1307.3237



ALICE (**2012**) F. Noferini QM2012

Typical flow result!

So : "Flow-like phenomena" are also seen in pp and pA, therefore:

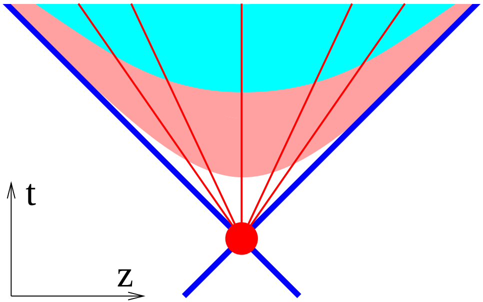
Heavy ion approach

= primary (multiple) scattering
+ subsequent fluid evolution

becomes interesting for pp and pA

5 (Pre)hadrons and secondary interactions

Primary interactions (red point) amout to multiple Pomeron exchanges, done in momentum space

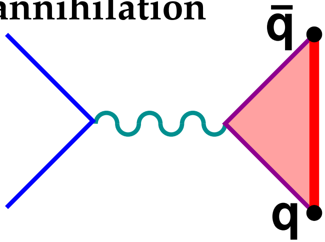


Each cut Pomeron corresponds to a parton ladder

We need it's space-time ($\eta_s - \tau$) evolution to construct an initial condition for a collective expansion

5.1 From partons to strings

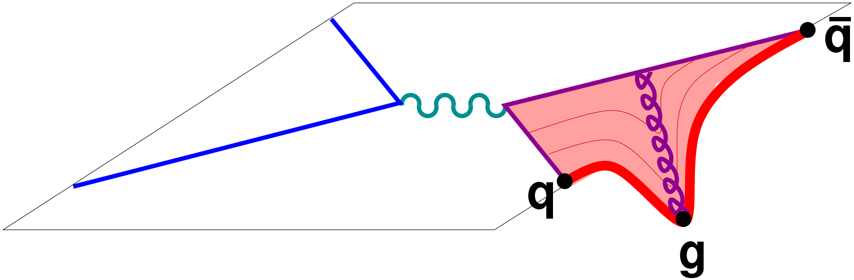
Electron-positron annihilation



Color field between two color charges
=> relativistic string

B. Andersson, G. Gustafson, G. Ingelman, and T. Sjostrand, Phys. Rep. 97 (83) 31
X. Artru, Phys. Rep. 97 (83) 147

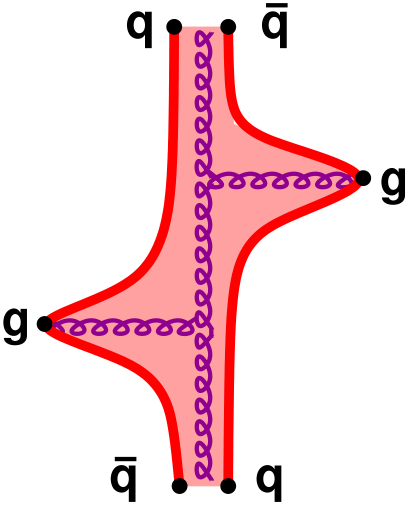
High pt gluon emission in e^+e^-



Kinky relativistic string

Cut Pomerons

(cut parton ladders)



Two kinky relativistic strings (at least)

Theoretical framework: **Classical string theory**

Nambu, Scherk, Rebbi ... 1969-1975

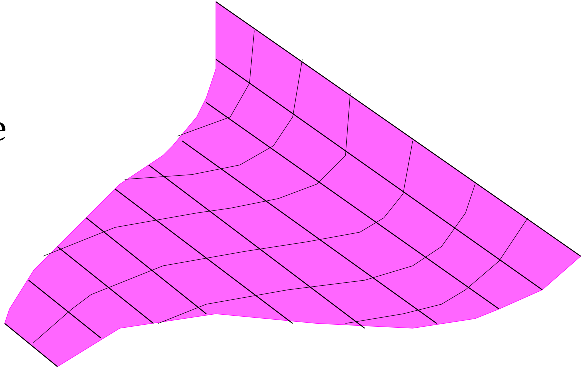
reviewed in PR 232, pp 87-299, 1993, PR 350, pp 93-289, 2001

String:

two-dimensional surface

$$x(\sigma, \tau)$$

in Minkowski space



Action $S = \int L d\tau d\sigma$

The Lagrangian is obtained by demanding **gauge invariance** of the action => Nambu-Goto Lagrangian:

$$L = -\kappa \sqrt{|\det g|}$$

with κ being the string tension, and with the metric

$$g_{ij} = \frac{\partial x^{\mu}}{\partial \xi^{i}} \frac{\partial x_{\mu}}{\partial \xi^{j}}$$

(using $\xi_1 = \sigma$, $\xi_2 = \tau$).

Gauge invariance:

$$g_{ij} = \frac{\partial x^\mu}{\partial \xi^i}\frac{\partial x_\mu}{\partial \xi^j} = \frac{\partial \xi'^m}{\partial \xi^i}\,\frac{\partial x^\mu}{\partial \xi'^m}\,\frac{\partial x_\mu}{\partial \xi'^n}\,\frac{\partial \xi'^n}{\partial \xi^j}$$

so (with M being Jacobien of $\xi'(\xi)$):

$$g_{ij} = M_{mi} g'_{mn} M_{nj} \;\to g = M^T g' M$$

So which gives

$$\sqrt{|\det g|} = \sqrt{|\det g'|}|\det M|$$

Using $\sqrt{|\det g|} = \sqrt{|\det g'|}|\det M|$ and in addition

$$d^2\xi' = |\det M| d^2\xi,$$

we get

$$\sqrt{|\det g|}d^2\xi = \sqrt{|\det g'|}d^2\xi'$$

= gauge invariance!!

With “dot” and “prime” referring to the partial derivatives with respect to σ and τ :

$$g = \begin{pmatrix} x'x' & x'\dot{x} \\ \dot{x}x' & \dot{x}\dot{x} \end{pmatrix}$$

we get

$$L = -\kappa\sqrt{|\det g|} = -\kappa\sqrt{(x'\dot{x})^2 - x'^2\dot{x}^2}$$

Euler-Lagrange equations of motion:

$$\frac{\partial}{\partial\tau}\frac{\partial L}{\partial\dot{x}_\mu} + \frac{\partial}{\partial\sigma}\frac{\partial L}{\partial x'_\mu} = 0.$$

We use the gauge fixing

$$x'^2 + \dot{x}^2 = 0 \text{ and } x'\dot{x} = 0,$$

which provides a very simple equation of motion, namely a wave equation,

$$\frac{\partial^2 x_\mu}{\partial \tau^2} - \frac{\partial^2 x_\mu}{\partial \sigma^2} = 0,$$

with the boundary conditions:

$$\partial x_\mu / \partial \sigma = 0,\ \sigma = 0, \pi.$$

Solution

$$x^\mu(\sigma,\tau) = \frac{1}{2}\left[f^\mu(\sigma+\tau) + f^\mu(\sigma-\tau) + \int_{\sigma-\tau}^{\sigma+\tau} g^\mu(\xi)d\xi\right].$$

We have

$$x^\mu(\sigma,\tau=0) = f^\mu(\sigma)$$

and

$$\dot{x}^\mu(\sigma,\tau=0) = g^\mu(\sigma)$$

Strings are classified according to the functions f and g.
We take $f^\mu = 0$ (no initial extension)

We also consider only strings with a

- □ **piecewise constant initial velocity g, which are called kinky strings.**

- □ **This string is characterized by a sequence of σ intervals $[\sigma_k, \sigma_{k+1}]$, and the corresponding constant values (say v_k) of g in these intervals.**

An electron-positron event (or a parton ladder) represents a **sequence of partons** of the type $q - g ... - g - \bar{q}$, with soft "end partons" q and $\bar{q}$, and hard inner gluons g.

The mapping "partons →string" is done such that we **identify a parton sequence with a kinky string**

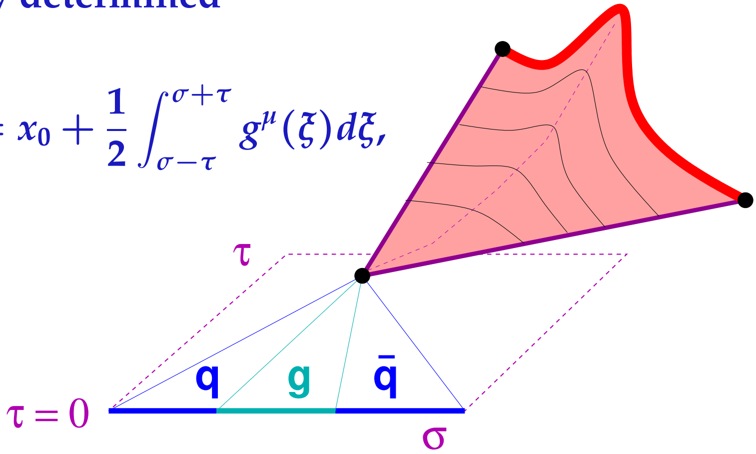
by requiring "parton = kink",

with $\sigma_{k+1} - \sigma_k =$ **energy of parton** k

and $v_k =$ **momentum of parton** k **/** E_k**.**

String evolution completely determined

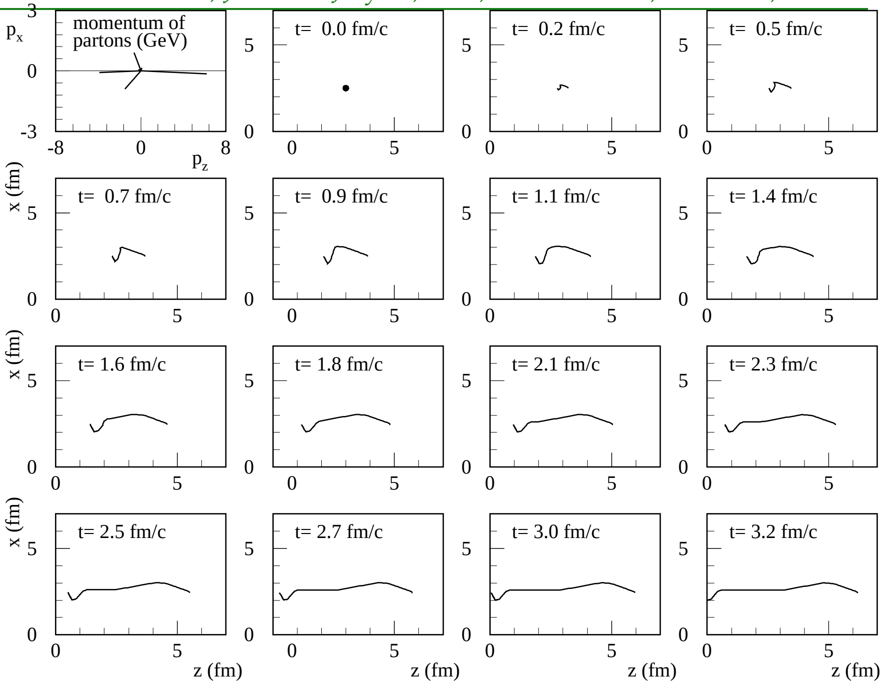
$$x^\mu(\sigma,\tau) = x_0 + \frac{1}{2}\int_{\sigma-\tau}^{\sigma+\tau} g^\mu(\xi)\,d\xi,$$



Mapping partons => string initial conditions

In the following figure,

we show the evolution of a string
generated in electron-positron annihilation
(4 internal kinks).


momentum of
partons (GeV)
p_x
p_z
3
0
-3
-8
0
8
t= 0.0 fm/c
t= 0.2 fm/c
t= 0.5 fm/c
t= 0.7 fm/c
t= 0.9 fm/c
t= 1.1 fm/c
t= 1.4 fm/c
t= 1.6 fm/c
t= 1.8 fm/c
t= 2.1 fm/c
t= 2.3 fm/c
t= 2.5 fm/c
t= 2.7 fm/c
t= 3.0 fm/c
t= 3.2 fm/c
x (fm)
z (fm)
0
5

5.2 Hadron production

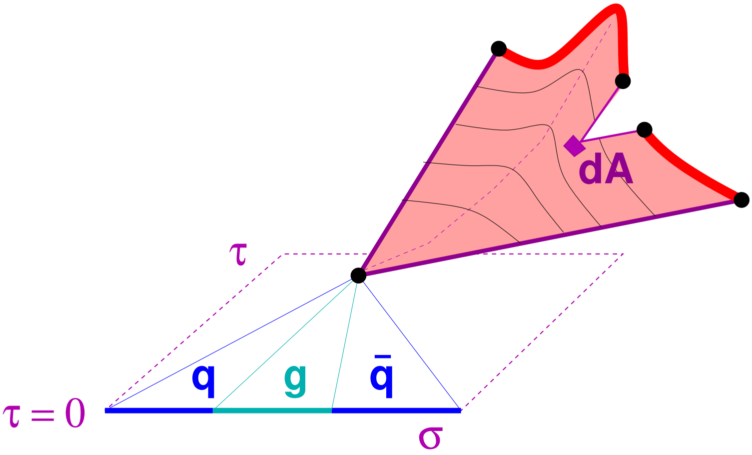
is finally realized via string breaking, such that string fragments are identified with hadrons.

Hypothesis: the string breaks within an infinitesimal area dA on its surface with a probability which is proportional to this area,

$$dP = p_B \, dA,$$

where p_B is the fundamental parameter of the procedure. [1]

[1] Elegant realization, making use of the dynamics of strings with piecewise constant initial conditions.


dA
τ
q
g
q̄
τ = 0
σ

A string break is realized via **quark-antiquark** or **diquark-antidiquark** pair production with probability

$$p_{i(j)} = \frac{1}{Z} \exp\left(-\pi \frac{M_{i(j)}^2}{\kappa}\right)$$

with

$$M_{ij} = M_i + M_j + c_i c_j M_0$$

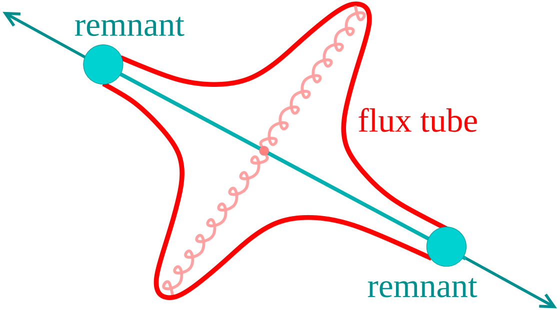
Transverse momenta $\vec{p}_t$ and $-\vec{p}_t$ are generated at each breaking, according to

$$f(k) \propto e^{-|\vec{p}_t|/2\bar{p}_t}, \tag{1}$$

with a parameter $\bar{p}_t$.

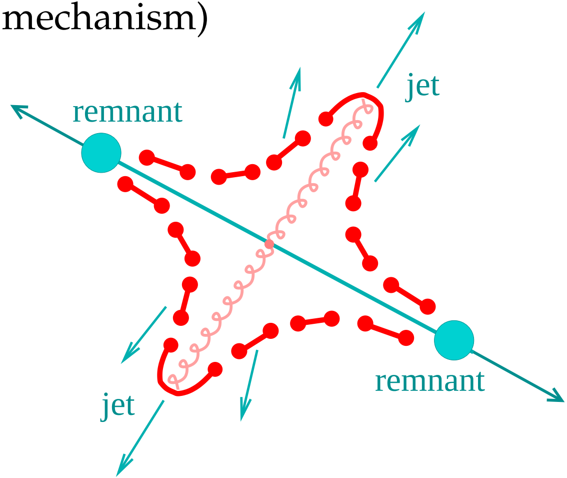
Jets:

Parton ladder = color flux tubes = **kinky strings**



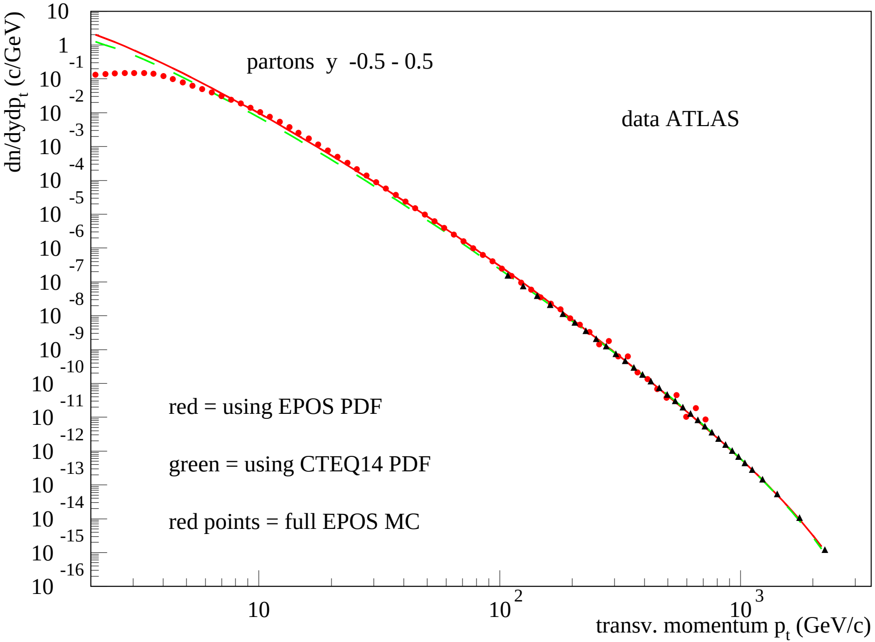
(here no IS radiation, only hard process producing two gluons)

which expand and break
via the production of quark-antiquark pairs
(Schwinger mechanism)

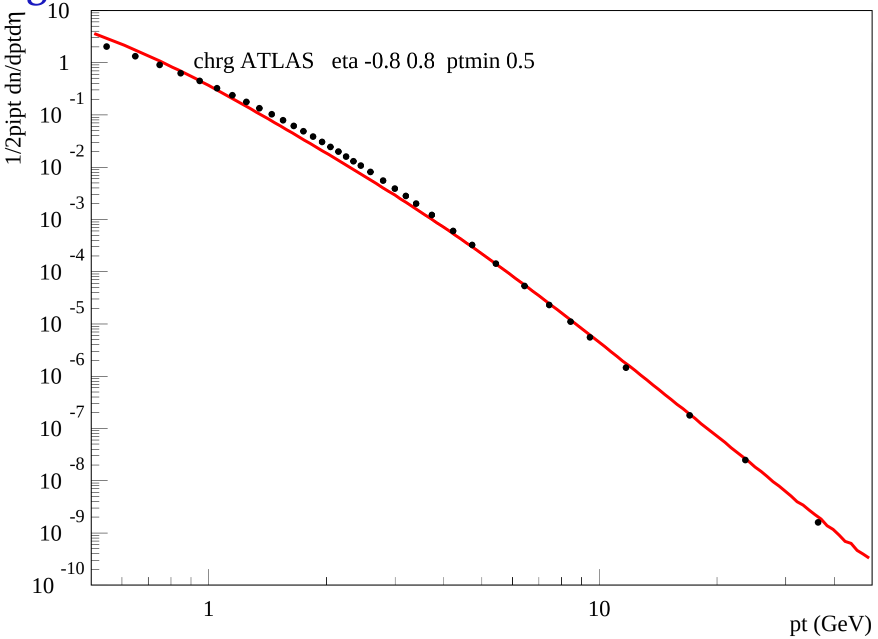


String segment = hadron. Close to "kink": jets

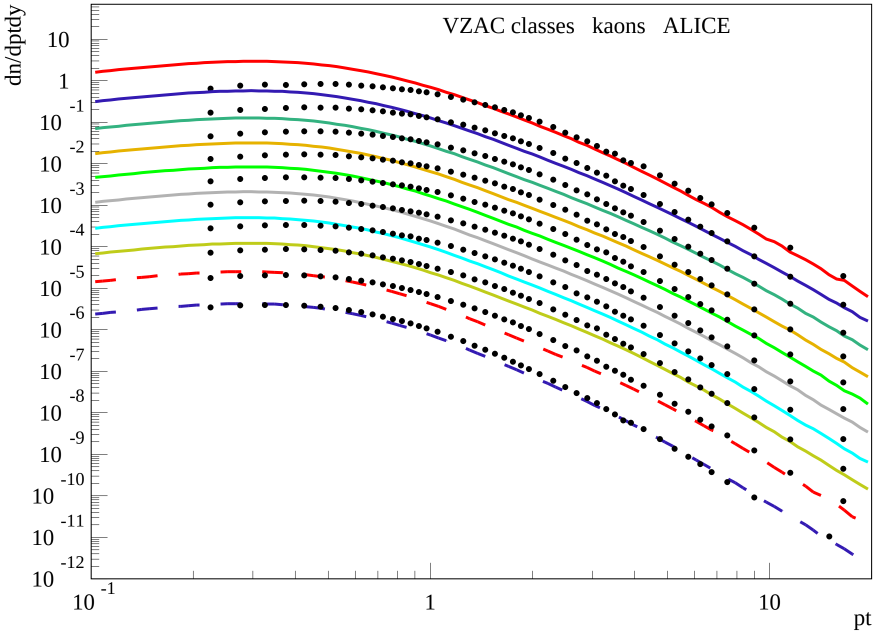
Example pp at 13 TeV : Partons



Charged hadrons ... too low around 2-3 GeV/c



Kaons diffent centralities ... not really great



5.3 Core-corona procedure

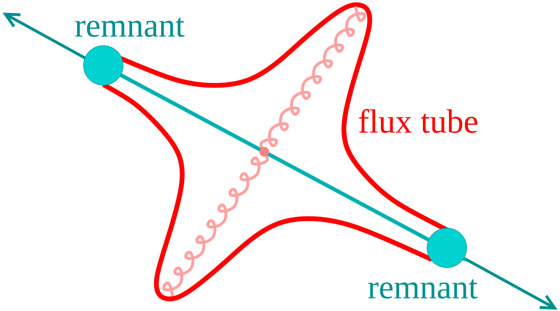
In case of multiple Pomerons (almost always)

- the standard procedure has to be modified, since the density of strings will be so high that they cannot possibly decay independently

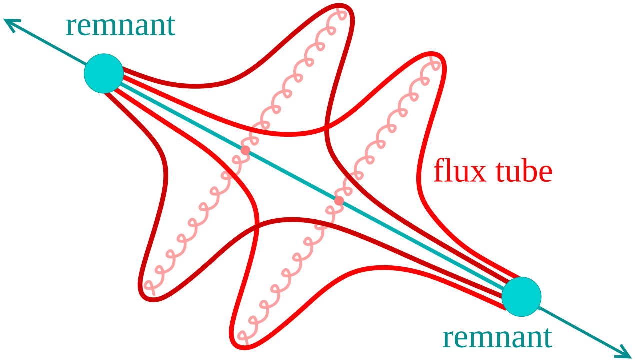
Some string pieces (pre-hadrons) will constitute bulk matter, others show up as jets

These are the same strings (all originating from hard processes at LHC) which constitute BOTH jets and bulk !

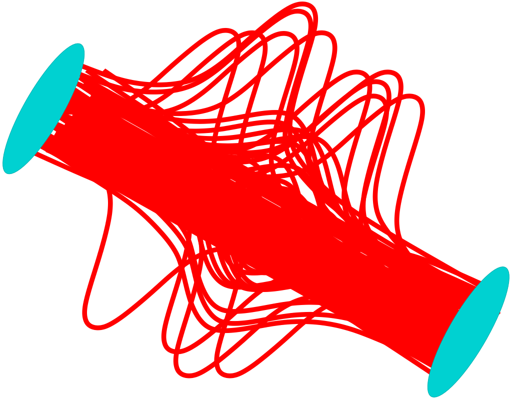
again: single scattering => 2 color flux tubes



... two scatterings => 4 color flux tubes



... many scatterings (AA) => many color flux tubes

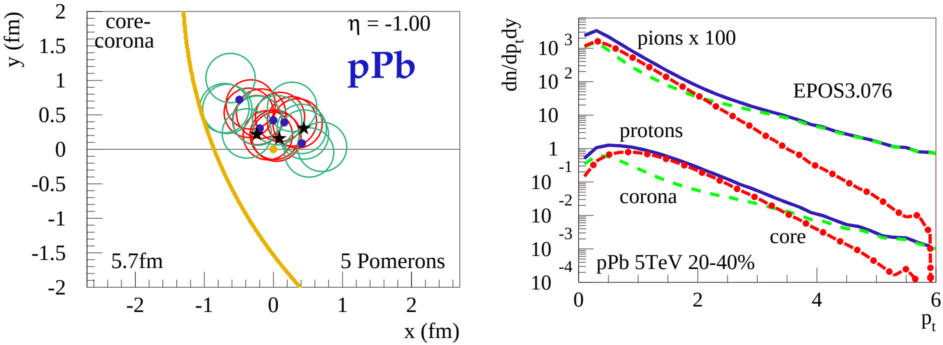


=> matter + escaping pieces (jets)

Core-corona procedure (for pp, pA, AA):

Pomeron => parton ladder => flux tube (kinky string)

String segments with high pt escape => **corona** the others form the **core** = **initial condition for hydro** depending on the local string density

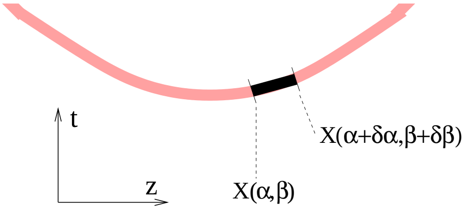


Core:

(we use α and β rather than σ and τ)

We split each string into a sequence of string segments, corresponding to widths $\delta\alpha$ and $\delta\beta$ in the string parameter space

Picture is schematic: the string extends well into the transverse dimension, correctly taken into account in the calculations



Energy momentum tensor and the flavor flow vector at some position x at initial proper time $\tau = \tau_0$:

$$T^{\mu\nu}(x) = \sum_i \frac{\delta p_i^\mu \delta p_i^\nu}{\delta p_i^0} g(x - x_i),$$

$$N_q^\mu(x) = \sum_i \frac{\delta p_i^\mu}{\delta p_i^0} q_i \, g(x - x_i),$$

$q \in u, d, s$: net flavor content of the string segments

$\delta p = \left\{ \frac{\partial X(\alpha,\beta)}{\partial \beta} \delta\alpha + \frac{\partial X(\alpha,\beta)}{\partial \alpha} \delta\beta \right\}$: four-momenta of the segments.

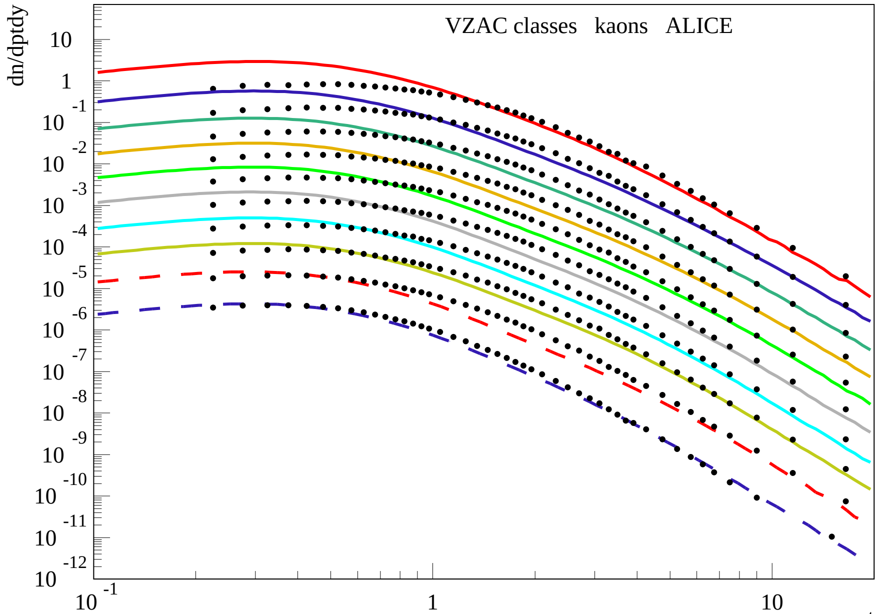
g: Gaussian smoothing kernel with a transverse width $\sigma_\perp$

The Lorentz transformation into the comoving frame provides the energy density ε and the flow velocity components v^i.

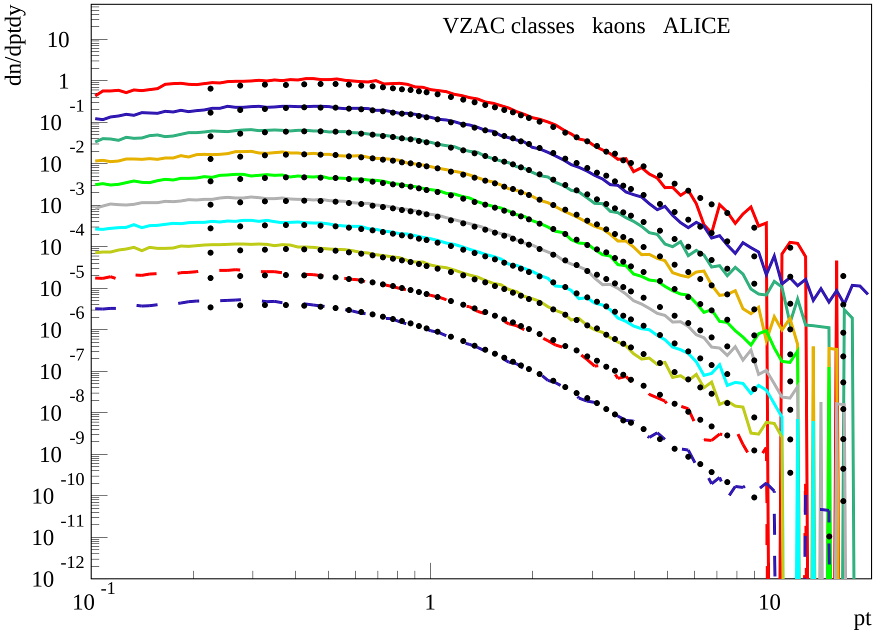
5.4 Some results sensitive to flow

- **Spectra**
- **correlations**

Kaons diffent centralities ... w/o core corona

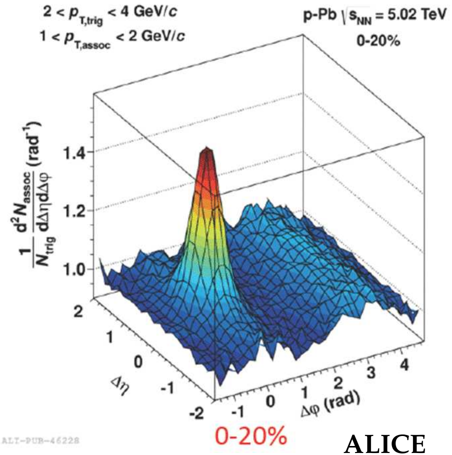


Kaons diffent centralities ... full simulation

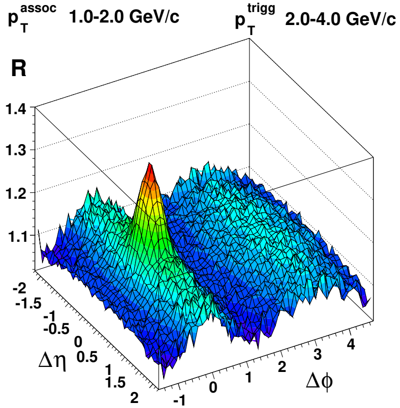


"Ridges" in pA

ALICE, arXiv:1212.2001, arXiv:1307.3237

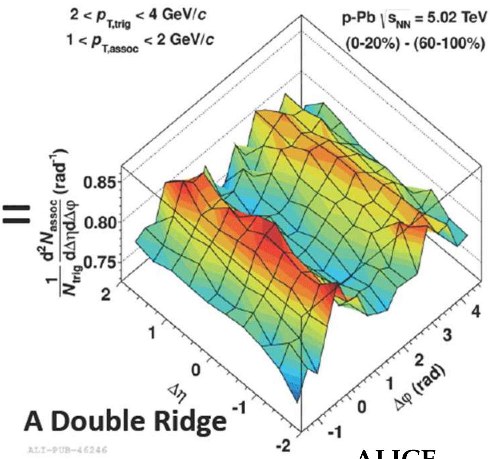


ALICE

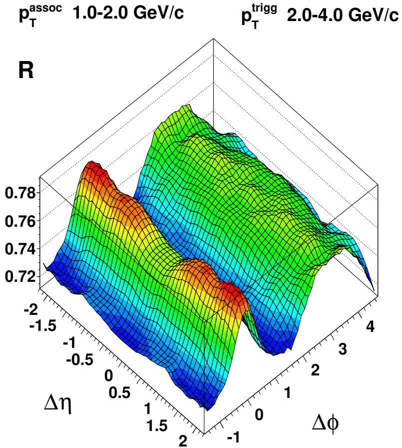


EPOS3.074

Central - peripheral (to get rid of jets)

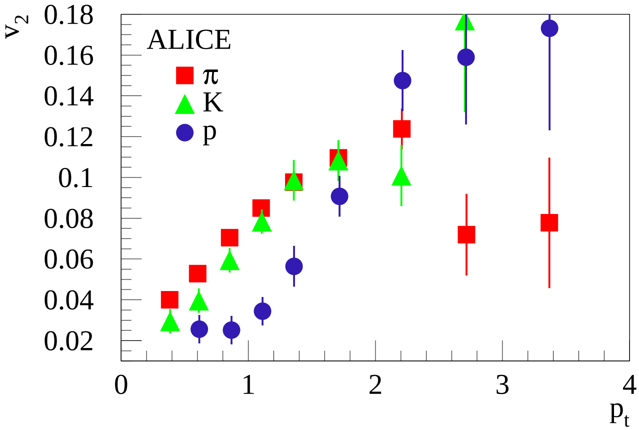


ALICE



EPOS3.074

Identified particle v2

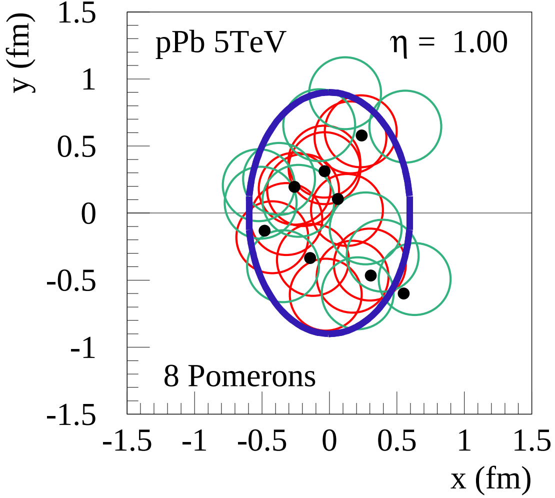


mass splitting, as in PbPb !!!

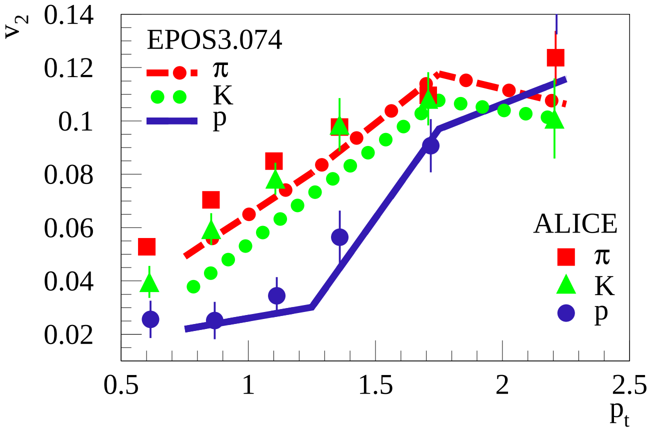
pPb in EPOS3:

Pomerons (number and positions) characterize geometry **(P. number ∝ multiplicity)**

random
azimuthal
asymmetry
=>
asymmetric flow
seen at higher pt for
heavier ptls



v2 for ß, K, p clearly differ

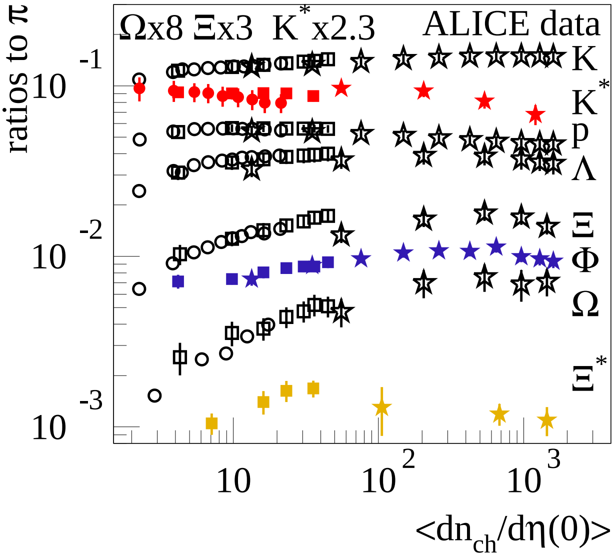


mass splitting, due to flow

5.5 Statistical particle production

Statistical particle production (from plasma decay) is very different from particle production via string decay

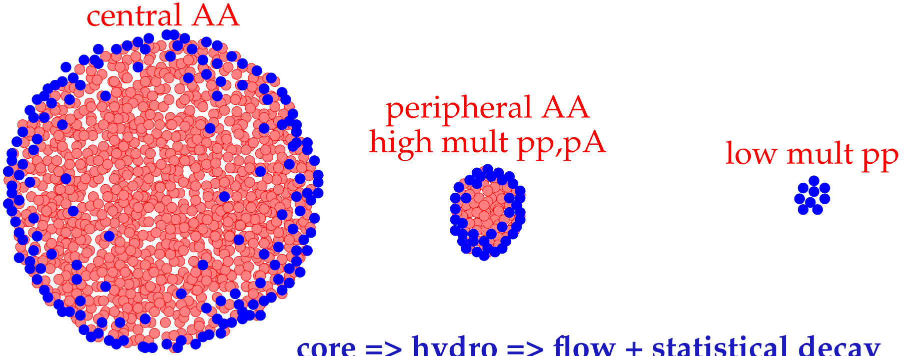
Particle ratios to pions vs $\left\langle \frac{dn_{ch}}{d\eta}(0) \right\rangle$



Core-corona picture in EPOS

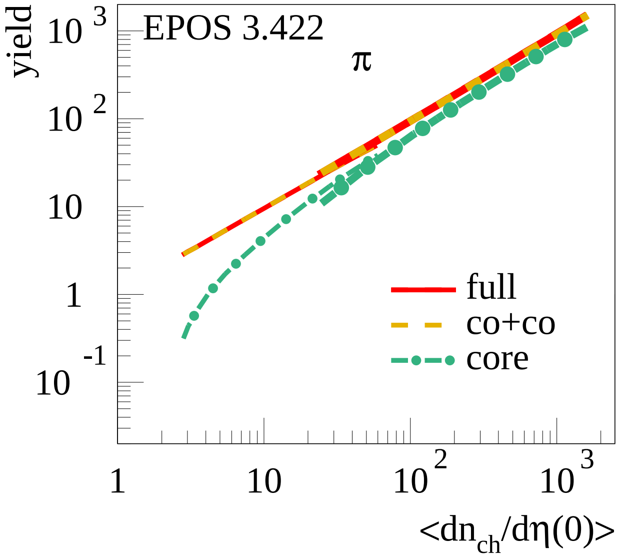
Phys.Rev.Lett. 98 (2007) 152301, Phys.Rev. C89 (2014) 6, 064903

Gribov-Regge approach => (Many) kinky strings
=> core/corona separation (based on string segments)



core => hydro => flow + statistical decay
corona => string decay

Pion yields: core & corona contribution



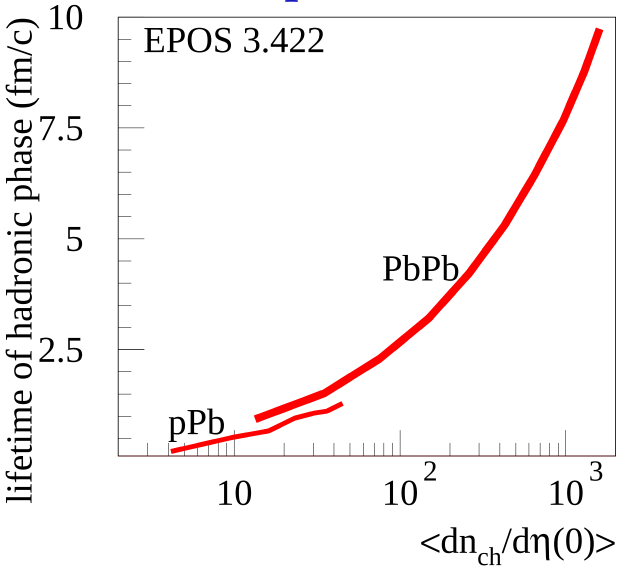
thin lines
= pp (7TeV)

intermediate lines
= pPb (5TeV)

thick lines
= PbPb (2.76TeV)

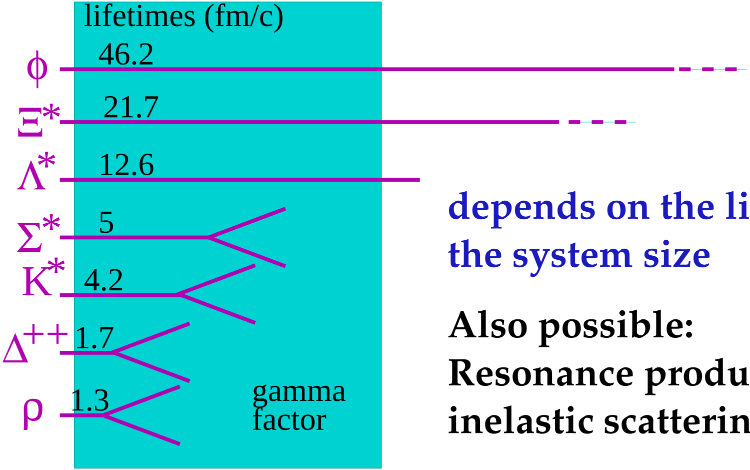
full = with hadronic cascade (UrQMD)

Lifetime of hadronic phase



Resonance suppression

in the hadronic stage (in-medium decay)

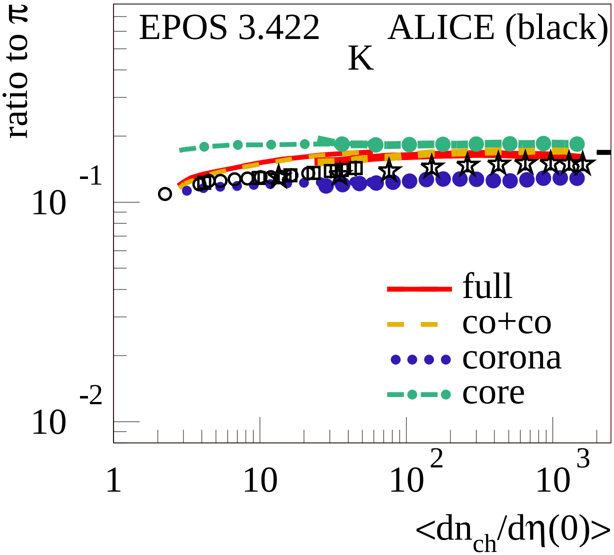


depends on the lifetime and the system size

Also possible:
Resonance production, inelastic scattering

but there is more

Kaon to pion ratio

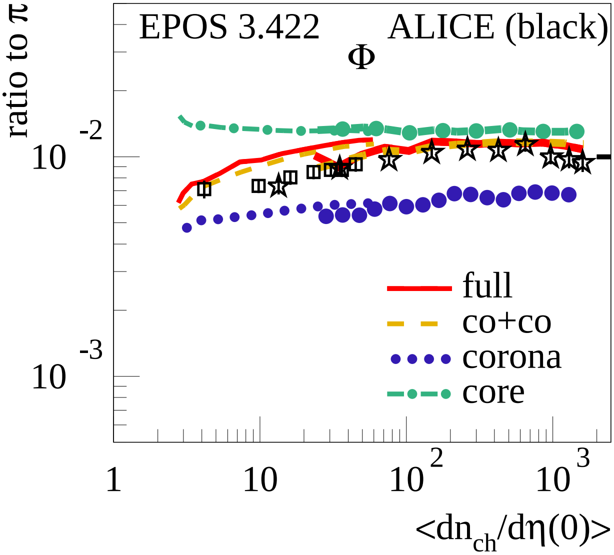


core hadronization:
$T = 164\,\mathrm{MeV},\ \mu_B = 0$

statistical model fit
(horizontal black line)
A. Andronic et al.,
arXiv:1611.01347
$T = 156.5\,\mathrm{MeV}, \mu_B = 0.7\,\mathrm{MeV}$

thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)
circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

Phi to pion ratio

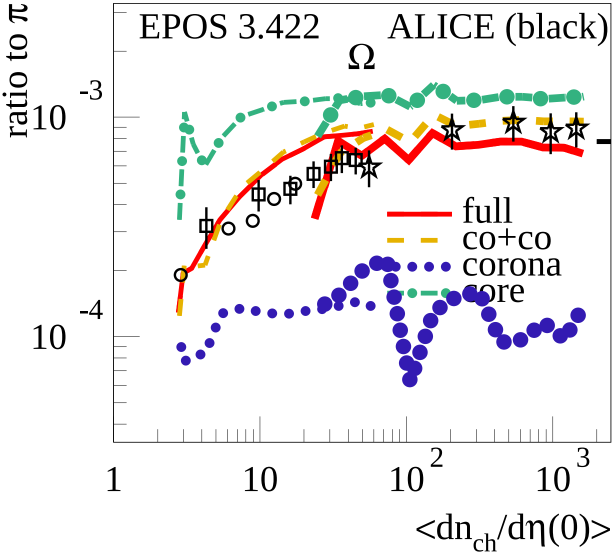


long-lived

$\tau \approx 46.2\,\mathrm{fm}/c$

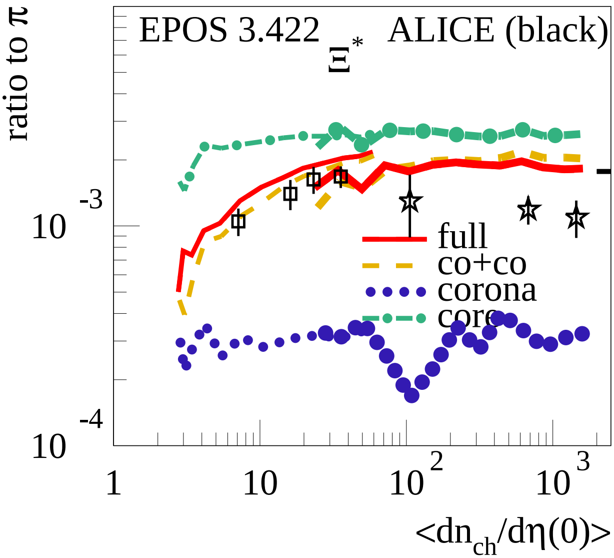
thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)
circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

Omega to pion ratio



thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)
circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

Ξ^* to pion ratio



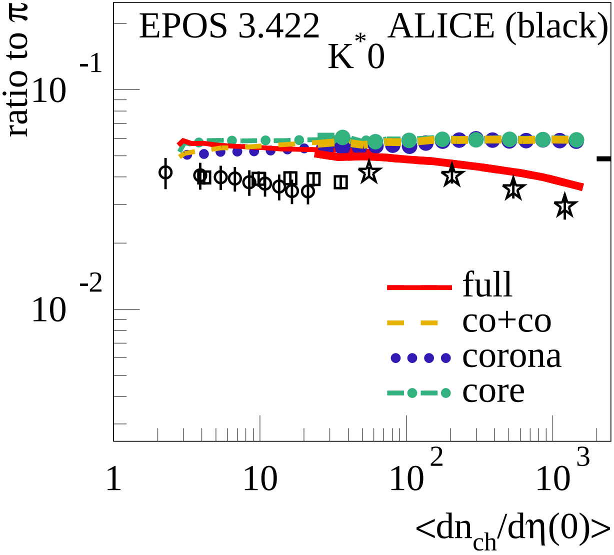
long-lived

$\tau \approx 21.7\,\mathrm{fm}/c$

thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

K^* to pion ratio



core ≈ corona

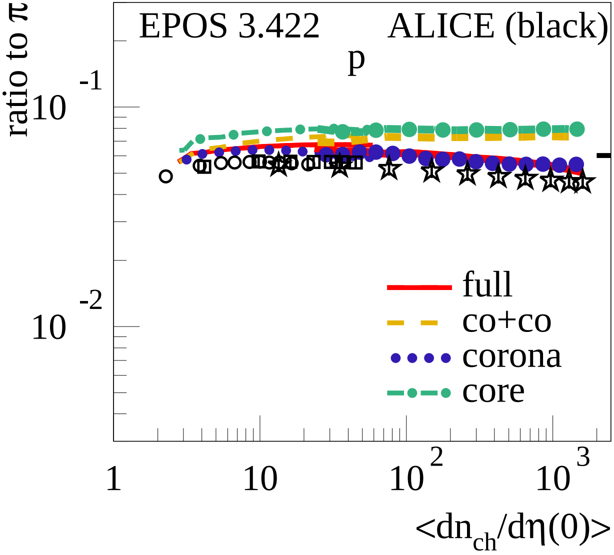
in-medium decay

$\tau \approx 4.2\,\mathrm{fm}/c$

thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

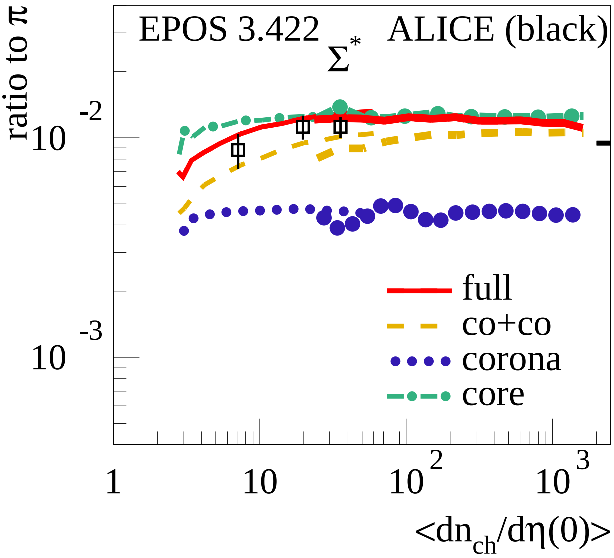
Proton to pion ratio



inelastic interactions (annihilation)

thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)
circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

Σ^* to pion ratio

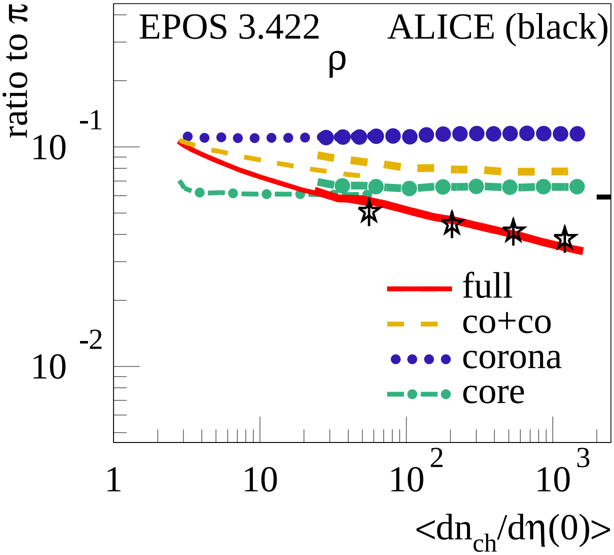


resonance production and in-medium decay

$\tau \approx 5\,\mathrm{fm}/c$

thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)
circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)

ρ to pion ratio



corona bigger !

in-medium decay

$\tau \approx 1.3\,\mathrm{fm}/c$

thin lines = pp (7TeV)
intermediate lines = pPb (5TeV)
thick lines = PbPb (2.76TeVVV)

circles = pp (7TeV)
squares = pPb (5TeV)
stars = PbPb (2.76TeV)